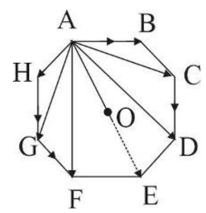
IIT JEE | MEDICAL | FOUNDATION

Solutions of Archive - JEE Main & Advanced

Introduction to Vector and Forces	Class - XI | Physics

JEE Main 2021

1.(B) $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}=8\overrightarrow{AO}=16\hat{i}+24\hat{j}-32\hat{k}$



2.(180) For this to happen $\sin\theta$ must be 0.

So $\theta=0°,\ 180°$ or $360°$; $0°$ and $360°$ are excluded

So $\theta=180°$

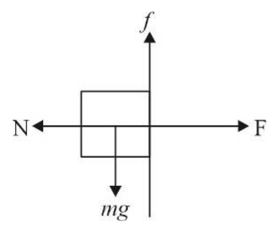
3.(25) For block to remain in equilibrium,

$mg=f$

$\therefore\quad mg=\mu N=\mu F$

$\therefore\quad 0.5(10)=(0.2)F$

$\therefore\quad F=25N$



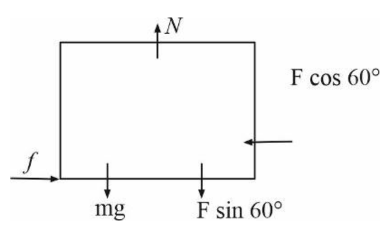
4.(C) $F\cos 60°=\mu(mg+F\sin 60°)$

$F\times\left(\frac{1}{2}-\frac{1}{3\sqrt{3}}\frac{\sqrt{3}}{2}\right)=\frac{1}{3\sqrt{3}}\times\sqrt{3}\times 10$

$F\left(\frac{2}{2\times 3}\right)=\frac{1}{3}\times 10$; $\quad F=10$

$F=3x=10$; $\quad x=10/3$

Rounding off $=3$



5.(A) $\overrightarrow{OA}=A\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$

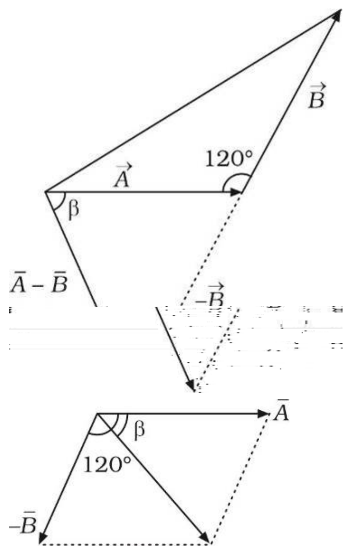
$\overrightarrow{OB}=A\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right)$

$\overrightarrow{OC}=A\left(-\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\right)$

$\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}=A\left[\frac{\sqrt{3}+1+\sqrt{2}}{2}\hat{i}+\frac{1-\sqrt{3}-\sqrt{2}}{2}\hat{j}\right]$

$\tan\alpha=\frac{1-\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}}$

6.(C)



$$\tan\beta = \frac{B\sin 120°}{A + B\cos 120°} = \frac{\sqrt{3}B/2}{A - B/2} = \frac{\sqrt{3}B}{2A - B}$$

7.(C) Along $Y \Rightarrow 10\sin 30° + 20\cos 30° + 15\cos 60° - 15\sin 45° - 20\sin 45°$

$$= 10\times\frac{1}{2} + 20\times\frac{\sqrt{3}}{2} + 15\times\frac{1}{2} - 15\times\frac{1}{\sqrt{2}} - 20\times\frac{1}{\sqrt{2}}$$

$$= 5 + 10\sqrt{3} + 7.5 - \frac{15}{\sqrt{2}} - 10\sqrt{2}$$

$$= 5 + 10\times 1.7 + 7.5 - 7.5\sqrt{2} - 10\sqrt{2}$$

$$= 5 + 17 + 7.5 - 7.5\times 1.4 - 14$$

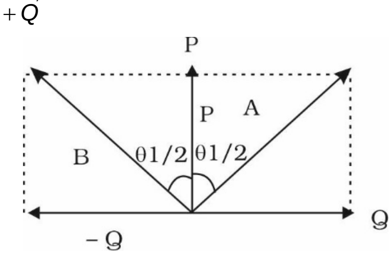
$$= 5 + 17 + 7.5 - 10.5 - 14 \;\; = 5\hat{j}$$

Along $x \rightarrow 10\cos 30° + 20\sin 30° - 15\sin 60° - 15\cos 45° + 20\cos 45° \;\; = 9.25\hat{i}$

$\bar{R} = 9.25\hat{i} + 5\hat{j}$

8.(A) Both statement I and statement II are true.

9.(C) $\vec{A} = \vec{P} + \vec{Q}$



$\vec{A} = \vec{B} = \vec{P} + \vec{Q} + \vec{P} - \vec{Q} \;=\; 2\vec{P}$

$\left|\vec{A} + \vec{B}\right| = \sqrt{3\left(P^2 + Q^2\right)} = \left|2\vec{P}\right|$

$3\left(P^2 + Q^2\right) = 4P^2$

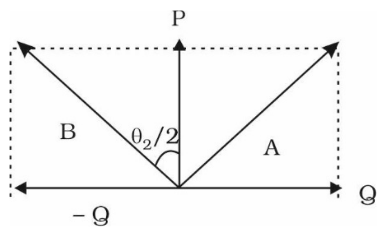
$\boxed{P = \sqrt{3Q}}$

From diagram $\tan\frac{\theta_1}{2} = \frac{Q}{P} = \frac{Q}{\sqrt{3}Q} = \frac{1}{\sqrt{3}} = \tan 30°$

$\frac{\theta_1}{2} = 30° \Rightarrow \theta_1 = 60°$

Second case

$\sqrt{2(P^2+Q^2)} = 2P$



$2P^2 + 2Q^2 = 4P^2$

$\boxed{P = Q}$

From diagram

$\tan\frac{\theta_2}{2} = \frac{Q}{P} = 1 = \tan 45°$

$\frac{\theta_2}{2} = 45° \Rightarrow \theta_2 = 90°$

$\theta_2 > \theta_1$

10.(82) Angle made by line AC with the vertical is 35°

So, component will be $P\cos 35°$

= 100× 0.819

= 81.9

11.(5) Net speed of summer along river flow is zero

$10\cos 120° + V_R = 0$

$V_R = 5\, m/s$

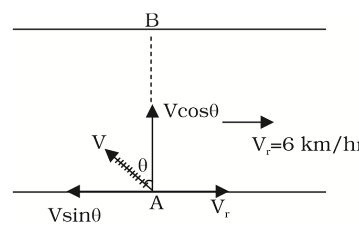
12.(120) $\vec{V}_{s,e} = \vec{V} + \vec{V}_r$

Swimmer will reach directly opposite to his starting point,

When $v\sin\theta = v_r$

or, $12\sin\theta = 6$

$\Rightarrow \quad \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$



$\therefore$ Swimmer should try to swim w.r.t direction of flow of river at an angle $= 30° + 90° = 120°$

13.(A) $|\vec{P}+\vec{Q}| = n|\vec{P}-\vec{Q}|$

$\Rightarrow \quad \sqrt{P^2+P^2+2P.P\cos\theta} = n\sqrt{P^2+P^2-2P.P\cos\theta}$

$\Rightarrow \quad 1+\cos\theta = n^2(1-\cos\theta) \quad \Rightarrow \quad \cos\theta(n^2+1) = n^2-1$

$\Rightarrow \quad \cos\theta = \frac{n^2-1}{n^2+1} \quad \therefore \quad \theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

14.(B) For (i) $\vec{A}+\vec{C} = \vec{B} \Rightarrow \vec{B}-\vec{A}-\vec{C} = 0$

For (ii) $\vec{A}+\vec{B}+\vec{C} = 0$

$\Rightarrow \vec{A}+\vec{B} = -\vec{C}$

For (iii)

$\vec{C}+\vec{B} = \vec{A}$

$\Rightarrow \vec{A}-\vec{C}-\vec{B} = 0$

For (iv)

$\vec{A}+\vec{B} = \vec{C}$

$\Rightarrow \vec{C}-\vec{A}-\vec{B} = 0$

15.(D) $2\sin\theta/2 = (2A\cos\theta/2)n$ (given)

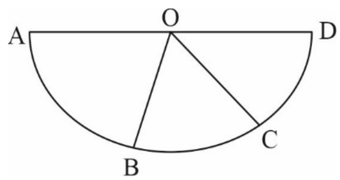
where $|\vec{x}| = |\vec{y}| = A$ (assumption)

$n = \tan\theta/2$

$$\cos\theta = \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} = \frac{1-n^2}{1+n^2}$$

16.(B) Assertion (A)

Taking LHS



$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$\overrightarrow{AD} = 2\overrightarrow{AO}$

Adding $= 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$ (RHS)

Reason $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$ (polygon law)

17.(C) $\vec{A}.\frac{\vec{B}}{|\vec{B}|} = \left(\hat{i} + \hat{j} + \hat{k}\right).\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$= \frac{2}{\sqrt{2}} = \sqrt{2}$

18.(3) p moves along $\vec{A}\times\vec{B} = \hat{i} - \hat{j} + \hat{k}$

Q moves along $\vec{A}\times\vec{C} = 2\hat{k}$

$$\cos\theta = \left(\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}\right).\left(\hat{k}\right) = \frac{1}{\sqrt{3}}$$

19.(B) $u = \frac{d}{t_1}$ = velo. of boy

$v = \frac{d}{t_2}$ = velo. of escalator

$\because \quad u + v = \frac{d}{t} \quad \Rightarrow \quad \frac{d}{t_1} + \frac{d}{t_2} = \frac{d}{t}$

$\Rightarrow \quad \frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} \quad \therefore \quad t = \frac{t_1 t_2}{t_1 + t_2}$

20.(D) $\vec{A}.\vec{B} = \left|\vec{A}\times\vec{B}\right|$

$\Rightarrow \quad \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = \left|\vec{A}\right|\left|\vec{B}\right|\left|\sin\theta\right|$

$\Rightarrow \quad \tan\theta = 1$

$\Rightarrow \quad \theta = 45°$

So

$\left|\vec{A}-\vec{B}\right| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{A^2 + B^2 - \sqrt{2}AB}.$

21.(C) $\bar{V}_{Bw} = \bar{V}_B - \bar{V}_w$

$\bar{V}_B = \bar{V}_{Bw} + \bar{V}_w$

$= 4\hat{i} + 4\hat{j} + \left(-\hat{j}\right)$

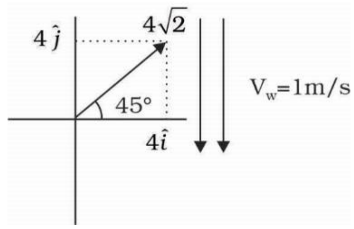
$= 4\hat{i} + 3\hat{j}$

$\bar{s} = \bar{V}_B \times t$

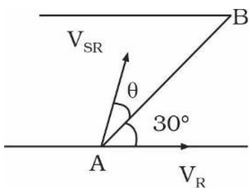
$= \left(4\hat{i} + 3\hat{j}\right) \times 3$

$= 12\hat{i} + 9\hat{j}$

$\left|\bar{s}\right| = \sqrt{144 + 81} = 15m$



22.(30)



For A to B

$\bar{V}_{SR} = \bar{V}_R$ $\therefore$ angle should be same $\theta = 30°$

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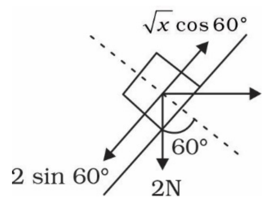
1.(C) $\vec{A} \times \vec{A} = A.A \sin 0° = 0$

2.(B) $\dfrac{|\hat{A}-\hat{B}|}{|\hat{A}+\hat{B}|} = \dfrac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} = \tan \theta/2$

3.(B) $\vec{\tau} = \vec{r} \times \vec{F} = \left(2\hat{i} + \hat{j} + 2\hat{k}\right) \times \left(3\hat{i} + 4\hat{j} - 2\hat{k}\right) \quad = -10\hat{i} + 10\hat{j} + 5\hat{k}$

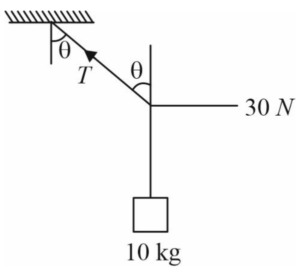
4.(12)



$2\sin 60° = \sqrt{x}\cos 60°$

$2\left(\dfrac{\sqrt{3}}{2}\right) = \sqrt{x}\left(\dfrac{1}{2}\right); \quad x = 12$

5.(3)



$T\sin\theta = 30\,; \qquad T\cos\theta = 100$

So, $\tan\theta = \dfrac{30}{100} \Rightarrow \theta = \tan^{-1}\left(3 \times 10^{-1}\right)$

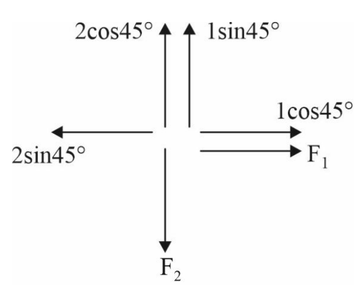
6.(C) $|\vec{A}| = |\vec{B}|; \qquad |\vec{A}+\vec{B}| = 2|\vec{A}-\vec{B}|$

$\Rightarrow \quad A^2 + B^2 + 2AB\cos\theta = 4(A^2 + B^2 - 2AB\cos\theta)$

$\Rightarrow \quad 1 + \cos\theta = 4(1-\cos\theta)$

$5\cos\theta = 3 \text{ or } \cos\theta = \dfrac{3}{5}$

7.(3) $R_x = 0 \Rightarrow F_1 + \dfrac{1}{\sqrt{2}} = \dfrac{2}{\sqrt{2}}$

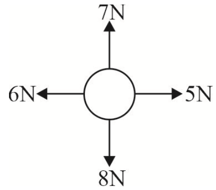


$F_1 = \dfrac{1}{\sqrt{2}}$

$R_y = 0 \Rightarrow F_2 = \dfrac{2}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \dfrac{3}{\sqrt{2}}$

$\dfrac{F_1}{F_2} = \dfrac{1}{3}; \qquad x = 3$

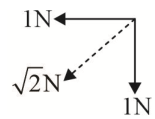
8.(A) The resultant of given four forces



$= \sqrt{2}N$ (at an angle) 135° from + ve

Direction of x-axis

So we have to add same amount of force in opposite direction so that there should be no acceleration



So $F_{req} = \sqrt{2}$ (at angle 45°)

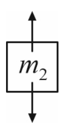
9.(2) Component of $\vec{A}$ on $\vec{B} = \dfrac{\vec{A} \bullet \vec{B}}{|\vec{B}|}$

$$\Rightarrow \quad \frac{(2\hat{i} + 3\hat{j} - \hat{k}) \bullet (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1^2 + 2^2 + 2^2}} \quad = \frac{2 + 6 - 2}{3} = 2$$

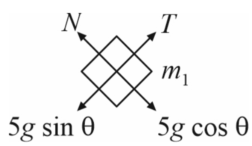
10.(5) Dot product is zero.

$2 + 8 - 2\alpha = 0$

$\alpha = 5$

11.(B) 

$T = 30N$



No friction as surface is smooth.

So, $5g \sin\theta = 30 \quad \Rightarrow \quad \sin\theta = \dfrac{30}{50}$

So, $\cos\theta = \dfrac{40}{50}$

Thus $N = 5g\cos\theta = 5 \times 10 \times \dfrac{40}{50} = 40N$

12.(C) $\vec{F} = 5\hat{i} + 3\hat{j} - 7\hat{k}, \vec{r} = 2\hat{i} + 2\hat{j} + \hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F} \quad = -17\hat{i} + 19\hat{j} - 4\hat{k}$

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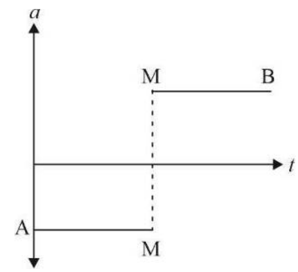
1.(C) Slope of velocity – time graph gives acceleration, as $\vec{a} = \frac{d\vec{v}}{dt}$.

For the part AM in given graph-slope is negative and constant. This means acceleration has a steady negative value.
For the part MB in given graph-slope is positive and constant. This means acceleration has a steady positive value.



2.(C) $v^2 = u^2 + 2aL$ ... (1)

$v_m^2 = u^2 + 2a\left(\frac{L}{2}\right) \Rightarrow aL = v_m^2 - u^2$ ... (2)

By (1) and (2), $v^2 = u^2 + 2v_m^2 - 2u^2$

$\Rightarrow \quad 2v_m^2 = v^2 + u^2 \qquad \therefore \quad v_m = \sqrt{\frac{u^2 + v^2}{2}}$

3.(C) Let height of the building is h.
and total time for the second stone is t.

$\frac{1}{2}gt^2 = h - 25$ ....(i)

$\frac{1}{2}g(t+1)^2 = h$ ....(ii)

Solving (i) & (ii) $h = 45\ m$
The question can also be solve odd number rule.

4.(D) $v = a_1 t_1$

$0 = a_1 t_1 - a_2 t_2$

$a_1 t_1 = a_2 t_2$

$\frac{t_1}{t_2} = \frac{a_2}{a_1}$

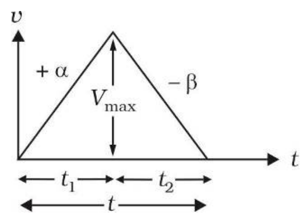
5.(D) $a = \frac{-\alpha}{m}x^2$

$\Rightarrow \quad \frac{vdv}{dx} = \frac{-\alpha}{m}x^2 \qquad \Rightarrow \quad \int_{v_0}^{0} v\,dv = \frac{-\alpha}{m}\int_0^x x^2 dx$

$\Rightarrow \quad \frac{-v_0^2}{2} = \frac{-\alpha}{m}\left(\frac{x^3}{3}\right) \qquad \Rightarrow \quad x = \left(\frac{3mv_0^2}{2\alpha}\right)^{1/3}$

Note that `m' is missing in each option. We should go for the option closest to the answer.

6.(A) acc = slope of v-t curve



$\Rightarrow \qquad \alpha = \dfrac{V_{max}}{t_1} \Rightarrow t_1 = \dfrac{V_{max}}{\alpha}$

Similarly, $-\beta = -\dfrac{V_{max}}{t_2} \Rightarrow t_2 = \dfrac{V_{max}}{\beta}$

$\because \qquad t = t_1 + t_2$

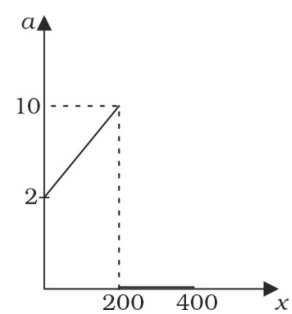
$\Rightarrow \qquad t = V_{max}\left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right)$

$\therefore \qquad V_{max} = \dfrac{\alpha\beta t}{\alpha + \beta}$

Total distance = Area under $|v| - t$ curve = $\dfrac{1}{2} \times V_{max} \times t = \dfrac{\alpha\beta t^2}{2(\alpha + \beta)}$

7.(B) $a = v\dfrac{dv}{dx}$

From x = 0 to x = 200 m



$\dfrac{dv}{dx} = \dfrac{50-10}{200} = \dfrac{4\cancel{0}}{20\cancel{0}} = \dfrac{1}{5}$

As $\qquad v = \dfrac{x}{5} + 10$

So, $\qquad a = v\dfrac{dv}{dx} = \left(\dfrac{x}{5} + 10\right)\dfrac{1}{5} = \dfrac{x}{25} + 2$

From $\qquad x = 200m$ to $x = 400m$

$\dfrac{dv}{dx} = 0 \qquad \therefore \qquad a = 0$

8.(A) Total distance $= 5 + 5 \times \dfrac{81}{100} \times 2 + 5 \times \left(\dfrac{81}{100}\right)^2 \times 2 + \ldots\ldots$

$$= 5 + 10 \times \frac{81}{100}\left[1 + \frac{81}{100} + \left(\frac{81}{100}\right)^2 + \ldots\ldots\right] = \frac{405}{19}m$$

Total time $= 1 + \dfrac{9}{10} \times 2 + \left(\dfrac{9}{10}\right)^2 \times 1 \times 2 + \ldots\ldots = 1 + 2 \times \dfrac{9}{10}\left[1 + \left(\dfrac{9}{10}\right) + \left(\dfrac{9}{10}\right)^2 + \ldots\ldots\right] = 19\ \Delta$

Average speed $= \dfrac{405/19}{19} = 2.5\ m/s$

9.(D) $v = v_0 + gt + Ft^2$

$v = \dfrac{dx}{dt}$

$\displaystyle\int_0^x dx = \int_0^1 (v_0 + gt + Ft^2)\, dt \qquad ; \qquad x = v_0 + \frac{g}{2} + \frac{F}{3}$

10.(D) $\upsilon = u + at$ (straight line positive slope)

$S = ut + \dfrac{1}{2}at^2$ (upward parabola)

11.(C) From the graph,

$$\frac{dV}{dx} = \frac{-V_0}{x_0}$$

Equation of line is,

$$V - V_0 = \frac{-V_0}{x_0}(x-0) \qquad \therefore\ V = \left(\frac{-V_0}{x_0}\right)x + V_0 \qquad \text{and, } a = \frac{VdV}{dx}$$

$$\therefore\ a = \left(\left(\frac{-V_0}{x_0}\right)x + V_0\right)\left(\frac{-V_0}{x_0}\right) \qquad \therefore\ a = \left(\frac{-V_0}{x_0}\right)^2 x - \frac{V_0^2}{x_0} \qquad \therefore\ \text{Slope} = \left(\frac{V_0}{x_0}\right)^2 \Rightarrow \text{ positive}$$

y-intercept $= \frac{-V_0^2}{x_0} \Rightarrow$ negative $\therefore$ Option is (3)

12.(No option matching)

$$\vec{V} = 0.5t^2\hat{i} + 3t\,\hat{j} + 9\hat{k} \Rightarrow \frac{d\vec{r}}{dt} = 0.5t^2\hat{i} + 3t\,\hat{j} + 9\hat{k} \Rightarrow d\vec{r} = (0.5t^2\hat{i} + 3t\,\hat{j} + 9\hat{k})dt$$

Assuming that mosquito starts at $t = 0$ sec from origin, its co-ordinates at $t = 2\,\text{sec}$ will be

$$\int_{(0\hat{i}+0\hat{j}+0\hat{k})}^{\vec{r}} d\vec{r} = \int_0^2 \left(0.5t^2\hat{i} + 3t\,\hat{j} + 9\hat{k}\right)dt \Rightarrow \vec{r} = \frac{4}{3}\hat{i} + 6\,\hat{j} + 18\hat{k};\ |\vec{r}| \approx 19$$

Let $\vec{r}$ makes α angle with x-axis and β angle with y-axis.

Then, $\cos\alpha = \frac{\vec{r}.\hat{i}}{|\vec{r}|} \Rightarrow \cos\alpha = \frac{4/3}{19} = \frac{4}{57}$ and hence

$\tan\alpha \approx 14.23 \Rightarrow \alpha \approx \tan^{-1}(14.23)$; $\qquad \cos\beta = \frac{\vec{r}.\hat{j}}{|\vec{r}|} - = \frac{6}{19}$ and hence $\tan\beta \approx 3 \Rightarrow \beta = \tan^{-1}(3)$

13.(50) $h = 35t - \frac{1}{2}gt^2 = 35(t+3) - \frac{1}{2}g(t+3)^2$

$35t - 5t^2 = 35t + 105 - 5t^2 - 45 - 30t$

$30t = 60$; $\qquad t = 2\,\text{sec}$

$h = 35 \times 2 - \frac{1}{2} \times 10 \times 2^2 = 50m$

14.(12) $V = \sqrt{5000 + 24x}$

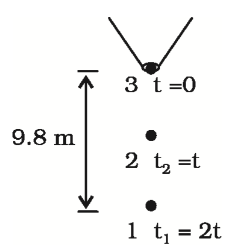
$\Rightarrow \qquad V^2 = 5000 + 24x \qquad \Rightarrow \qquad 2v\frac{dv}{dx} = 0 + 24 \Rightarrow a = 12m/s^2$

$a = 12m/s^2$

15.(B) For 1st drop; $\qquad s = \vec{u}t_1 + \frac{1}{2}\vec{a}t_1^2$

$9.8 = 0 + \frac{1}{2} \times 9.8 \times (2t)^2$

$\Rightarrow t = \frac{1}{\sqrt{2}}\,\text{sec}$



For second drop:

$s = \vec{u}t_2 + \frac{1}{2}\vec{a}t_2^2$; $\qquad h_2 = 0 + \frac{1}{2} \times 9.8 \times \left(\frac{1}{\sqrt{2}}\right)^2$

Or, $h_2 = 2.45m$

Height of 2nd drop from ground ; $= 9.8 - 2.45 = 7.35m$

16.(1) $v^2 = 20 + 2x \quad \Rightarrow \quad 2v\dfrac{dv}{dx} = 2$

$\therefore \quad a = v\dfrac{dv}{dx} = 1m/s^2$

17.(D) n = no. of droplet coming per sec.

Time spacing between two consecutive droplet $= \dfrac{1}{n}$sec

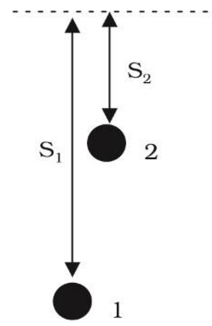
At t = 4 sec.

*Particle 1 has travelled for t = 4 sec.

$s_1 = \dfrac{1}{2} \times 9.8 \times 4^2$

*particle 2 has travelled for $t = \left(4 - \dfrac{1}{n}\right)\text{sec} \Rightarrow s_2 = \dfrac{1}{2} \times 9.8\left(4 - \dfrac{1}{n}\right)^2$

$\because s_1 - s_2 = 34.3 \quad \Rightarrow 4.9 \times 4^2 - 4.9\left(4 - \dfrac{1}{n}\right)^2 = 34.3 \Rightarrow n = 1$



18.(A) Differentiating both sides w.r.t t

$\dfrac{d}{dt}t = \dfrac{d}{dt}(mx^2 + nx) \quad \Rightarrow \quad 1 = (2mx + n)v$

$\Rightarrow \quad \dfrac{d}{dt}\dfrac{1}{v} = \dfrac{d}{dt}(2mx + n)$ (Differentiating again)

$\Rightarrow \quad -\dfrac{1}{v^2}\dfrac{dv}{dt} = 2mv \quad \Rightarrow \quad -\dfrac{dv}{dt} = r = 2mv^3$

19.(D) Applying relative analysis w.r.t. balloon

$u_r = 0 \qquad a_r = -g$

Distance between balloon are slope $= \left|\dfrac{1}{2}(g)t^2\right|$

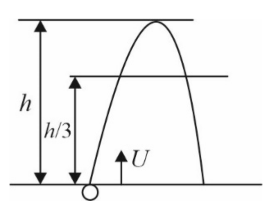
Now, w.r.t. ground

$-75 = 10t - \dfrac{1}{2}gt^2 \quad \Rightarrow \quad t = 5\,\text{sec}$

Thus, distance $= 5 \times (5)^2 = 125\,m$

20.(A) $ds = \alpha t\,dt + \beta t^2 dt$ **;** $\quad s = \dfrac{\alpha}{2}(4-1) + \dfrac{\beta}{3}(8-1) = \dfrac{3\alpha}{2} + \dfrac{7\beta}{3}$

21.(B) $U = \sqrt{2gh}$



$\dfrac{h}{3} = \sqrt{2gh}\,t - \dfrac{1}{2}gt^2$

$gt^2 - 2\sqrt{2gh}\,t + \dfrac{2h}{3} = 0$

$$t = \frac{2\sqrt{2gh} \pm \sqrt{8gh - 4g \times \frac{2h}{3}}}{2g} = \frac{2\sqrt{2gh} \pm \sqrt{\frac{16gh}{3}}}{2g} = \frac{2\sqrt{2gh} \pm 4\sqrt{\frac{gh}{3}}}{2g}$$

$$\frac{t_1}{t_2} = \frac{2\sqrt{2gh} - 4\sqrt{\frac{gh}{3}}}{2\sqrt{2gh} + 4\sqrt{\frac{gh}{3}}} = \frac{2\sqrt{2} - \frac{4}{\sqrt{3}}}{2\sqrt{2} + \frac{4}{\sqrt{3}}} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

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Kinematics of a Particle	Class - XI | Physics

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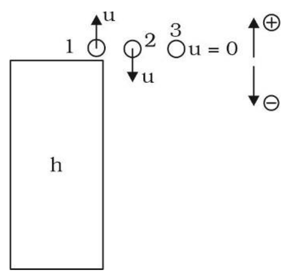
1.(3) $-h = ut_1 - \frac{1}{2}gt_1^2$ ...(i)

$-h = -ut_2 - \frac{1}{2}gt_2^2$ ...(ii)

$-h = -\frac{1}{2}gt_3^2$ ...(iii)

Solving (i), (ii) and (iii) we will get

$t_3 = \sqrt{t_1 t_2} = \sqrt{6 \times 1.5}$; $t_3 = 3\,\text{sec}$



2.(B) $\frac{t_{ascent}}{t_{descent}} = \sqrt{\frac{g-a}{g+a}}$

$= \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{8}{12}} = \sqrt{\frac{2}{3}} \Rightarrow t_{ascent} : t_{descent} = \sqrt{2} : \sqrt{3}$

3.(D) Differentiating x_P and x_Q with time

$U_P = \frac{dx_P}{dt} = \alpha + 2\beta t,\ V_Q = \frac{dx_Q}{dt} = f - 2t$

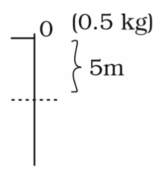
Equating both $t = \frac{f-\alpha}{2(1+\beta)}$

4.(5) $v = u + g(t)$

$10 = 10(t)$; $t = 1$

$s = \frac{1}{2} \times (10)(1) = 5\,\text{m}$

$\therefore$ height $= 5$ m



5.(6) $s_1 = 50 \times 2 - \frac{1}{2} \times 10 \times 2^2 = 80$

$v_1 = 50 - 10 \times 2 = 30\ m/s$

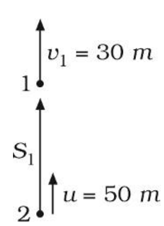
$a_{\text{rel}} = 0$

$\therefore$ In order to meet ball 1 and 2

$s_1 = (u - v_1) \times t'$

$\Rightarrow$ $80 = (50 - 30) \times t'$ $\Rightarrow$ $t' = 4s$

$t_{req} = 2 \times t' = 6s$



6.(18) $V_{avg} = \frac{x}{\frac{x}{3 \times 11} + \frac{x}{3 \times 22} + \frac{x}{3 \times 33}} = \frac{3}{\frac{1}{11} + \frac{1}{22} + \frac{1}{33}}$

$= \frac{3}{\frac{6+3+2}{66}} = \frac{3 \times 66}{11} = 18 m/s.$

7.(D) Position of Ball (A)

$y_1 = 180 - \frac{1}{2}gt^2$ . . . (i)

Position of Ball (B)

$y_2 = 180 - u(t-2) - \frac{1}{2}g(t-2)^2$ . . . (ii)

Balls meet at $y = 100$.

$\Rightarrow \quad 100 = 180 - 5t^2 \Rightarrow t = 4s$

Using in (ii) ; $\quad 100 = 180 - u(2) - 5(2)^2$

$2u = 60$; $\quad u = 30m / s$

8.(C) $u = 0$

$10 = \frac{1}{2}at^2$ ... (1)

$10 + x = \frac{1}{2}(a)(2t)^2$ ... (2)

$\frac{(2)}{(1)} \frac{10+x}{10} = 4$; $\quad x = 30m$

9.(3) Stopping distance $(s) = \frac{u^2}{2a}$

$\frac{S_2}{S_1} = \frac{u_2^2}{u_1^2}$ (also $u_2 = \frac{u_1}{3}$)

$\frac{S_2}{S_1} = \frac{1}{9}$; $\quad S_2 = \frac{27}{9} = 3$

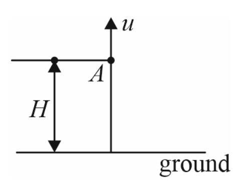
10.(100) $\frac{dv}{ds} = 5$; $\quad a = V\frac{dv}{ds}$

$a = 20 \times 5$; $\quad a = 100m / \sec^2$

11.(392) Ttime of flight =6 sec

$t = \frac{u + \sqrt{u^2 + 2gh}}{g} = \frac{19.6 + \sqrt{(19.6)^2 + 2 \times 98}}{9.8}$; $H = 6g$

Total height from ground $= H + \frac{u^2}{2g} = 8g = \frac{k}{5}$; $\quad k = 392$



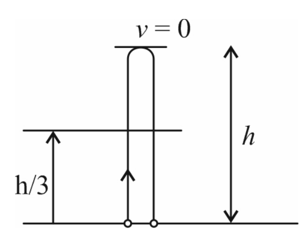
12.(B) By $v^2 = u^2 + 2as$; $\quad u = \sqrt{2gh}$

by $s = ut + \frac{1}{2}at^2$; $\quad \frac{h}{3} = \sqrt{2gh}\,t - \frac{1}{2}gt^2$

$\Rightarrow \quad 3gt^2 - 6\sqrt{2gh}\,t + 2h = 0$

$\Rightarrow \quad t_1 \, and \, t_2 = \frac{6\sqrt{2gh} \pm \sqrt{2gh - 24gh}}{6g}$

$t_1 \, and \, t_2 = \sqrt{\frac{2h}{g}} \pm \frac{2}{3}\sqrt{\frac{3h}{g}}$; $\quad \frac{t_1}{t_2} = \frac{\sqrt{2} - \sqrt{3}.\frac{2}{3}}{\sqrt{2} + \sqrt{3}.\frac{2}{3}} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$



13.(B) $t = \sqrt{x} + 4$

$\Rightarrow \quad x = (t-4)^2$

$\frac{dx}{dt} = 2(t-4) \quad \therefore \quad t = 4 \Rightarrow \frac{dx}{dt} = 0$

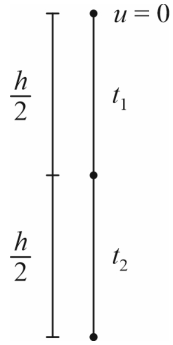
14.(D) Juggler throws n balls per second

So, 1 ball every $\frac{1}{n}$ second

When first ball reaches highest point, he throws second ball.

$\therefore \quad h = \frac{1}{2}gt^2 = \frac{1}{2} \times g \times \frac{1}{n^2} = \frac{g}{2n^2}$

15.(D) $\frac{h}{2} = \frac{1}{2}gt_1^2$



$\Rightarrow \quad t_1 = \sqrt{\frac{h}{g}}$

Also $\quad h = \frac{1}{2}g(t_1 + t_2)^2$

$\Rightarrow \quad \sqrt{\frac{h}{g}} = \frac{(t_1 + t_2)}{\sqrt{2}}$

$\Rightarrow \quad t_1 = \frac{t_1 + t_2}{\sqrt{2}}$

$\Rightarrow \quad t_1\left(\sqrt{2} - 1\right) = t_2$

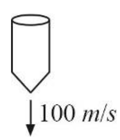
16.(A) Let time of fall $= t$

$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 19.6}{9.8}} = 2\,\text{sec}$

Required distance $d = \left(\frac{9km}{h}\right) \times (2\,\text{sec})$

$= (2.5\ m\ /\ s) \times (2\ \text{sec}) \quad = 5m$

17.(A) Taking upward direction as positive



$v = u + at$

$\Rightarrow \quad v = -100 - 10t$

$t = 10s,$

$v = -100 - 10 \times 10 = -200\ \text{m/s}$

18.(C) $v^2 = u^2 - 2as\ ; \qquad \left(\frac{u}{3}\right)^2 = u^2 - 2a \times 0.04$

$a = \frac{u^2 \times 8\ /\ 9}{2 \times 4 \times 10^{-2}}$

Now $v^2 = u^2 - 2as'\ ; \qquad 0 = u^2 - 2 \times u^2 \times \frac{8}{9 \times 8 \times 10^{-2}} \times s'$

$s' = \frac{9 \times 10^{-2}}{2} = 4.5\,\text{cm}\ ;\ s' = 4 + x = 4.5 \quad \therefore\ x = 0.5\,cm$

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1.(B) $y = \alpha x - \beta x^2$

$dy = \alpha dx - 2\beta x dx$

$\frac{dy}{dx} = \alpha - 2\beta x$

$\frac{dy}{dx} = 0$ at $x = \frac{\alpha}{2\beta}$

$y = \frac{\alpha\alpha}{2B} - \frac{\beta \times \alpha^2}{4\beta^2}$; $y = \frac{\alpha^2}{4\beta}$

2.(728) $\langle\omega\rangle = \frac{\omega_f + \omega_i}{2} = \frac{\theta}{t}$ $\therefore$ $\theta = \frac{2460+900}{2} \times 26 \times \frac{2\pi}{60}$ radian

Number of revolutions = $\frac{\theta}{2\pi} = 728$

3.(5) $|\Delta P| = m(\Delta V)_y = \frac{5}{1000}(10) = 5 \times 10^{-2} kg\ m/s$ $\therefore$ $x = 5$

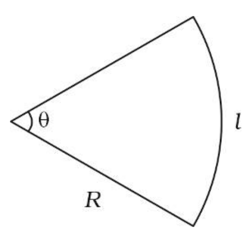
4.(C) $H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{(25)^2 \sin^2 45°}{2 \times 10} = 15.625m$ $\Rightarrow$ $t = \frac{u \sin\theta}{g} = \frac{25 \times \sin 45°}{10} = 1.77 \sec$

5.(D) In plane frame – bomb has $u_{\text{bomb, plane}} = 0$, with $a = g$ downward

6.(C) $1ly = 9.616 \times 10^{15} \times 10^{15} m$

$1Au = 1.5 \times 1d^1 m$

$l = R\theta$



$4.4 \times 9 : 46 \times 10^{15} = R\theta$

$\theta = 4s$

$= 4 \times 4.843 \times 10^{-6}$

$= 1.94 \times 10^{-5} rad$; $R\theta = \theta R$

$4.4 \times 9.46 \times 10^{15} = 1.94 \times 10^{-5} \times R$

$R = 2.1455 \times 10^{21} m$

Speed $= 8Au/s$

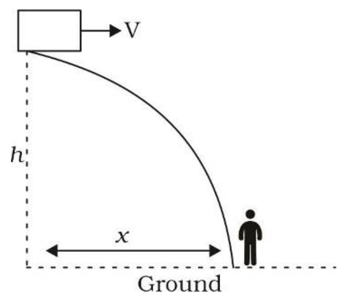
$= 8 \times 1.5 \times 10^{11} m/s = 12 \times 10^{11} m/s$

4 revolution means $\Rightarrow$ distance = $4 \times 2\pi R$

$time = \frac{distance}{speed} = \frac{4 \times 2\pi R}{12 \times 10^{11}} = \frac{8 \times 3.14 \times 2.14\pi \times 10^{21}}{12 \times 10^{11}}$

$t = 4.5 \times 10^{10}$ sec

7.(A) $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$



$x = Vt = V\sqrt{\frac{2h}{g}}$

Distance $= \sqrt{x^2 + h^2}$

$= \sqrt{\frac{2V^2h}{g} + h^2}$

8.(D) $\theta_1 = 42^0, \theta_2 = 48°$

Here θ_1 and θ_2 complementary angle

Hence, $R_1 = R_2$

But $H \propto \sin^2\theta$

so $H_2 > H_1$

9.(200) $< w > = \frac{\theta}{t} = \frac{w_f + w_i}{2}$; $\theta = \frac{10}{2}(1800 + 600) \times \frac{1}{60} rev$; $\theta = 200 rev$

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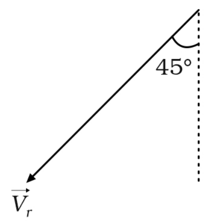
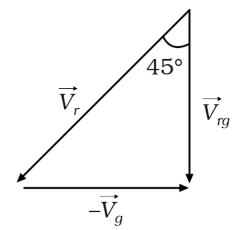
1.(C) $\vec{V}_{rg} = \vec{V}_r + \left(-\vec{V}_g\right)$

$\left|\vec{V}_g\right| = 15\sqrt{2}\,km\ /\ h$

$\therefore \left|\vec{V}_{rg}\right| = 15\sqrt{2}\,km\ /\ h = \dfrac{30}{\sqrt{2}}\,km\ /\ h$

As angle is 45°





2.(B) $\alpha = 6t^2 - 2t$; $\omega_0 = 10, \theta_0 = 4$

$\dfrac{d\omega}{dt} = 6t^2 - 2t$; $\displaystyle\int_{10}^{\omega} d\omega = 6\int_0^t t^2 dt - 2\int_0^t t\,dt$

$\omega - 10 = 2t^3 - t^2$; $\dfrac{d\theta}{dt} = 2t^3 - t^2 + 10$

$\displaystyle\int_4^{\theta} d\theta = 2\int_0^t t^3 dt - \int_0^t t^2 dt + 10\int_0^t dt$

$\theta - 4 = \dfrac{2t^4}{4} - \dfrac{t^3}{3} + 10t$; $\theta = 4 + \dfrac{t^4}{2} - \dfrac{t^3}{3} + 10t$

3.(D) $R = \dfrac{V^2(2\sin\theta\cos\theta)}{g}$

$t = \dfrac{V\sin\theta}{g} \Rightarrow V = \dfrac{gt}{\sin\theta}$ $\Rightarrow$ $R = \dfrac{g^2t^2}{\sin^2\theta}\cdot\dfrac{2\sin\theta\cos\theta}{g}$

$\tan\theta = \dfrac{2gt^2}{R} = \dfrac{20t^2}{R}$; $\cot\theta = \dfrac{R}{20t^2}$

4.(B) $\theta = \omega t + \dfrac{1}{2}\alpha t^2$ $\Rightarrow$ $5 = \dfrac{1}{2}(\alpha)(1)^2$ $\Rightarrow$ $\alpha = 10\,rad\ /\ s^2$

Angle rotated is one second

$= wt + \dfrac{1}{2}\alpha t^2$ $= 10(1) + \dfrac{1}{2}(10)(1)^2$ $= 10 + 5 = 15\,rad$

5.(20) Assume initial velocity be v ; Then $\vec{v} = \dfrac{v}{\sqrt{2}}\hat{i} + \dfrac{v}{\sqrt{2}}\hat{j}$

At 2 sec ; $\vec{v} = \dfrac{v}{\sqrt{2}}\hat{i} + \left(\dfrac{v}{\sqrt{2}} - 2g\right)\hat{j}$

Given $v = 20 = \sqrt{\left(\dfrac{v}{\sqrt{2}}\right)^2 + \left(\dfrac{v}{\sqrt{2}} - 2g\right)^2}$

Solving we get $v = \dfrac{40}{\sqrt{2}}$; $H_{max} = \dfrac{v^2\sin^2\theta}{2g} = \dfrac{\left(\dfrac{40}{\sqrt{2}}\dfrac{1}{\sqrt{2}}\right)^2}{20} = 20\,m$

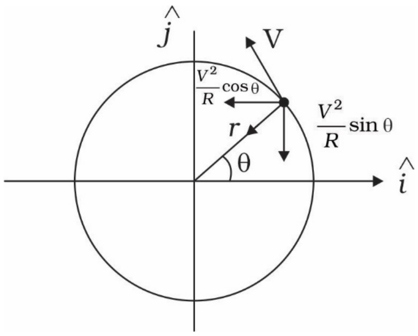
6.(2) $\overrightarrow{acc} = \dfrac{\vec{F}}{m} = 100\hat{i} + 50\hat{j}$

$$\frac{a}{b} = \frac{\frac{1}{2}(100)t^2}{\frac{1}{2}(50)t^2} = 2$$

7.(A) Ranges will be same if pojection angles are θ and $90 - \theta$

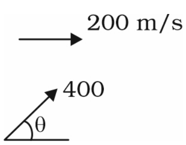
$\therefore$ so, $h_1 = \dfrac{u^2 \sin^2\theta}{2g}$, $h_1 = \dfrac{u^2 \sin^2(90-\theta)}{2g}$

8.(C) Centripetel acceleration will be $\dfrac{V^2}{r}$ towards center.



$\therefore$ Taking components of $\dfrac{V^2}{R}$; $\vec{a} = \dfrac{V^2}{R}\cos\theta(-\hat{i}) + \dfrac{V^2}{R}\sin\theta(-\hat{j})$

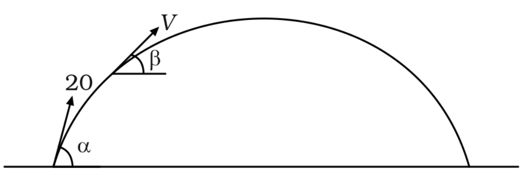
9.(60)



$200 = 400\cos\theta$

$\cos\theta = 1/2$; $\theta = 60°$

10.(B)



$V\sin\beta = 20\sin\alpha - 10\times10$ ...(i)

$V\cos\beta = 20\cos\alpha$ ...(ii)

From (i) and (ii), $\tan\beta = \tan\alpha - 5\sec\alpha$

11.(A) $x = 4\sin\left(\dfrac{\pi}{2} - \omega t\right) = 4\cos(\omega t)$

$y = 4\sin(\omega t)$

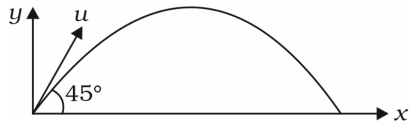
$\sin^2(\omega t) + \cos^2(\omega t) = \dfrac{x^2}{16} + \dfrac{y^2}{16} = 1$ $\Rightarrow x^2 + y^2 = 4^2$

Equation of circle.

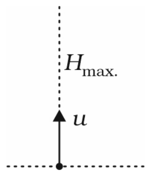
12.(B) $R_{max.} = \dfrac{u^2}{g} = 100m$

$H_{max.} = \dfrac{u^2}{2g} = \dfrac{1}{2}(100) = 50m$

Case I



Case II



13.(B) Distance covered $= x = R\theta$

$\Rightarrow \quad x = R\left(3\frac{\pi}{4}\right) \quad \Rightarrow \quad R = \frac{4x}{3\pi} = \frac{4 \times 60}{3\pi} = \frac{80}{\pi} m$

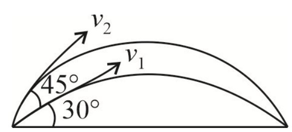
$\therefore \quad$ Displacement $= \sqrt{R^2 + R^2 - 2R^2 \cos(135°)}$

$= \sqrt{3.4R^2} = \sqrt{3.4\left(\frac{80}{\pi}\right)^2} \approx 47 m$

14.(D) $R = H_{max}$

$\frac{2u^2 \sin\theta \cos\theta}{g} = \frac{u^2 \sin^2\theta}{2g} \quad \Rightarrow \quad 4 = \tan\theta$

15.(C) $H_{max} =$ same ; T = same



$H = \frac{(v\sin\theta)^2}{2g} =$ constant

$v\sin\theta =$ same

$\Rightarrow \quad \frac{v_1}{v_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 45°}{\sin 30°} \quad \therefore \quad \frac{v_1}{v_2} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2} : 1$

16.(5) $y = 5x(1-x)$; $\quad y = 5x - 5x^2$

$y = x\tan\theta - \frac{1}{2}gx^2 \quad \Rightarrow \quad \tan\theta = 5,$

$u_x = u\cos\theta = 1$; $\quad u_y = u\sin\theta$

Divide $u_y = \tan\theta = 5$

17.(C) $R = \frac{u^2 \sin 2\theta}{g}$

According to questions $\theta_1 = 45°$; $\quad \theta_2 = 30°$

Range $\alpha \sin 2\theta$; $\quad \frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2} = \frac{\sin(2(45°))}{\sin(2(30°))} = \frac{\sin 90}{\sin 60} = \frac{2}{\sqrt{3}}$

18.(1) For first ball $\quad$ For second ball

$T = \frac{2u}{g} \quad T' = \frac{2u_y}{g}$

$h = \frac{u^2}{2g} \quad h' = \frac{u_y^2}{2g}$

If $T = T'$ $\quad$ i.e. $\quad u = u_y$

$\Rightarrow \quad h = h'$

19.(B) $\vec{r} = 3t\hat{i} + 5t^3\hat{j} + 7\hat{k}$

$\vec{v} = 3\hat{i} + 15t^2\hat{j} + 0\hat{k}$

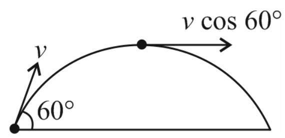
$\vec{a} = 30t\hat{j}$

at $t = 1$ sec ; $\vec{a} = 30\hat{j}$

20.(C) $KE = E = \frac{1}{2}mv^2$ @ projection

$KE = \frac{1}{2}m(v\cos 60°)^2$

$= \frac{E}{4}$ @ Maximum height

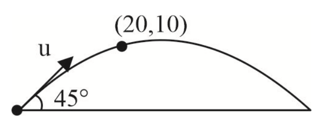


21.(15) $R_{max} = \frac{u^2}{g}$

Now $\frac{R_{max}}{2} = \frac{u^2 \sin 2\theta}{g}$

$\frac{u^2}{2g} = \frac{u^2}{g}\sin 2\theta \Rightarrow \theta = 15°$

22.(D) $y = x\tan\theta - \frac{gx^2}{2u\cos^2\theta}$; $10 = 20 - \frac{10 \times (20)^2}{2 \times u^2 \times \frac{1}{2}}$



$\frac{(20)^2}{u^2} = 1 \Rightarrow u = 20$ m/s ; $T = \frac{2u\sin 45°}{g} = \frac{2 \times 20}{10} \times \frac{1}{\sqrt{2}} = 2\sqrt{2}$

$\frac{T}{\sqrt{2}} = 2$ sec ; $V_x = u\cos 45° = \frac{20}{\sqrt{2}} = 10\sqrt{2}$

$v_y = u_y - gt = 10\sqrt{2} - 10 \times 2$ $\therefore$ $\vec{P} = mv_x\hat{i} + mv_y\hat{j} = 100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$

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Dynamics of a Particle	Class - XI | Physics

1.(25) Block will not slip if $f_{max} \geq mg \sin\theta$

$\therefore \quad \mu mg \cos\theta \geq mg \sin\theta$

$\therefore \quad \mu \geq \tan\theta \qquad$ and, $\tan\theta = \dfrac{dy}{dx}$

$\therefore \quad y = \dfrac{x^2}{4}; \qquad \dfrac{dy}{dx} = \dfrac{2x}{4} = \dfrac{x}{2}$

$\therefore \quad \mu \geq \tan\theta; \qquad 0.5 \geq \dfrac{x}{2}$

$\therefore \quad x \leq 1 \qquad \Rightarrow \qquad \sqrt{4y} \leq 1$

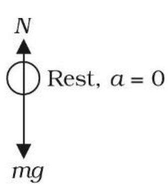
$\therefore \quad y \leq \dfrac{1}{4} m \qquad \therefore \qquad y = 25 cm$

2.(C) $F = \dfrac{K}{R^3} = \dfrac{mV_0^2}{R}; \qquad V_0$: orbital velocity

$\Rightarrow \quad V_0 = \sqrt{\dfrac{K}{mR^2}}$

Time of revolution; $T = \dfrac{2\pi R}{V_0} = \dfrac{2\pi R}{\sqrt{\dfrac{K}{mR^2}}} = 2\pi\sqrt{\dfrac{m}{K}} \cdot R^2 \qquad \therefore \qquad T \propto R^2$

3.(492)

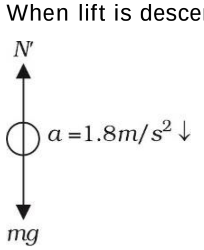


When lift is stationary

$N - mg = 0$

$N = mg = 60g \qquad \therefore \qquad m = 60 kg$

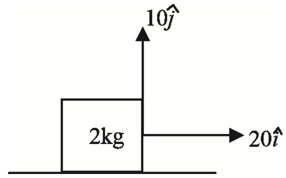
When lift is descending with acceleration $1.8 m/s^2$



$mg - N' = m \times 1.8$

$N' = m(g - 1.8) = 60(10 - 1.8) = 492 N$

4.(500) Along x-axis



$$a = \frac{20}{2} = 10 m/s^2$$

$$s = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 10 \times (10)^2 = 500 m$$

5.(C) Answer given by NTA would be correct if direction of F would be right wards

As F applied on B is towards left

$\therefore \quad a_{CM} = \frac{-F}{2M}$ (taking right direction as positive)

$a_B = +a$ (given) ; $\quad a_m = \frac{m_A a_A + m_B a_B}{m_A + m_B}$

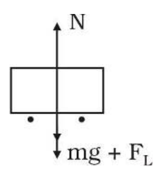
$\frac{-F}{2M} = \frac{Ma + Ma_B}{2M} \quad \therefore \quad a_B = -\frac{(F + Ma)}{M}$; $\quad |a_B| = \frac{F + Ma}{M}$

6.(12) $\vec{a} = \frac{\vec{F}}{m} = \left(\hat{i} + 1.5\hat{j} + 2.5\hat{k}\right)$; $\quad \vec{S} = \vec{R}_0 + \left(\vec{u}t + \frac{1}{2}\vec{a}t^2\right)$

$\vec{R}_0 =$ initial coordinate.

$\vec{S} = \frac{1}{2}\left(\hat{i} + 1.5\hat{j} + 2.5\hat{k}\right) \times 16 = 8\hat{i} + 12\hat{j} + 20\hat{k}$; $\quad b = 12$

7.(A) $\mu_S N = \frac{mv^2}{R}$ ... (1)

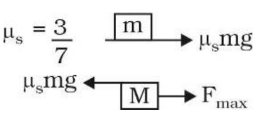


$mg + F_L = N$ ... (2)

$\Rightarrow \quad mg + F_L = \frac{mv^2}{\mu_S R}$

$\therefore \quad F_L = m\left(\frac{v^2}{\mu_S R} - g\right)$

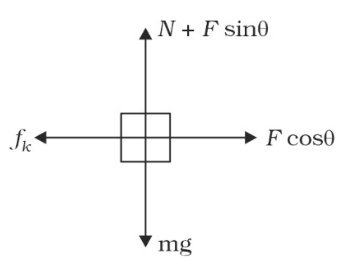
8.(21) $\mu_s = \frac{3}{7}$



$\because \quad a_m = a_M \text{ at } F_{max} \quad \Rightarrow \quad \frac{\mu_s mg}{m} = \frac{F_{max} - \mu_s mg}{M}$

$\therefore \quad F_{max} = \mu_s (M + m) g = \frac{3}{7} \times 5 \times 9.8 N = 21 N$

9.(D) $N = (mg - F \sin\theta)$



$f_K = \mu_K N$

$f_K = \mu_\kappa (mg - F\sin\theta)$

$\therefore \quad F\cos\theta - f_\kappa = ma$

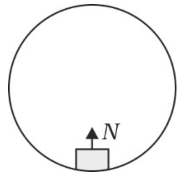
$\Rightarrow \quad a = \frac{F\cos\theta - \mu_\kappa (mg - F\sin\theta)}{m}$

$\Rightarrow \quad a = \frac{F}{m}\cos\theta - \mu_\kappa \left(g - \frac{F}{m}\sin\theta\right)$

10.(D) $N = m\omega^2 r$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{40}$

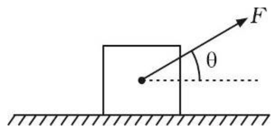
or $N = \left(\frac{200}{1000}\right)\left(\frac{\pi}{20}\right)^2 \times \left(\frac{20}{100}\right)$



$N = 9.859 \times 10^{-4} N$

11.(5) $F = \frac{\mu mg}{(\cos\theta + \mu \sin\theta)}$

$F_{min} = \frac{\mu mg}{\sqrt{1+\mu_2}} = \frac{\frac{1}{\sqrt{3}} \times 10}{\sqrt{\frac{4}{3}}} = 5$

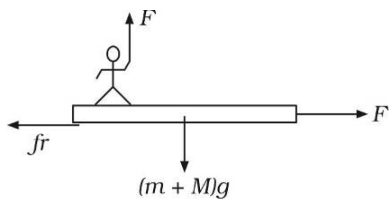


12.(30) $F \le \mu N$

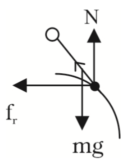
$F \le \mu\,[(m+M)g - F]$

$F \le \frac{\mu(m+M)g}{1+\mu}$

$F \le 30\ N$



13.(C)



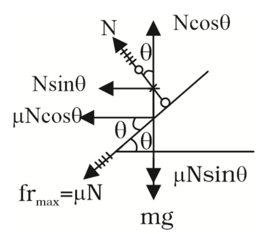
On flat rough circular track;

$fr_{max} = \frac{mV_{max}^2}{R} \Rightarrow \mu N = m\frac{V_{max}^2}{R}$

$\Rightarrow \mu mg = m\frac{V_{max}^2}{R} \Rightarrow V_{max} = \sqrt{\mu gR} = \sqrt{0.2 \times 9.8 \times 2} = 1.97 m/s = 7.12 km/hr$

So, if cyclist moves with 7 km/hr on this track, he will be safe.

On banked rough circular track:



$N\sin\theta + \mu N\cos\theta = \frac{mv_{max}^2}{R}$ ... (i)

$N\cos\theta - \mu N\sin\theta = mg$ ... (ii)

$\frac{(i)}{(ii)} \Rightarrow V_{max} = \sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)} = \sqrt{2 \times 9.8 \times \frac{(\tan 45° + 0.2)}{(1 - 0.2 \times \tan 45°)}} = 5.4 m/s = 19.52 km/hr$

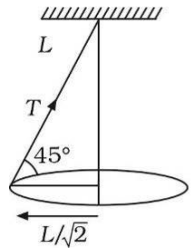
So on banked circular road, 18.5 km/hr speed will be safe.

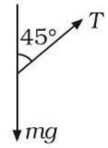
14.(C) $Ma = V.\frac{dm}{dt} - Mg$

$20{,}000 = 500.\frac{dm}{dt} - 10{,}000$

$\frac{dm}{dt} = 60 kg/\sec$

15.(B)

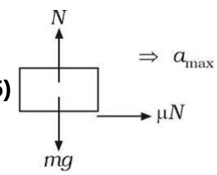




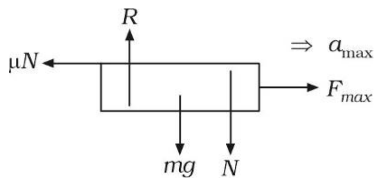
$\dfrac{T}{\sqrt{2}} = mg$; $\qquad \dfrac{T}{\sqrt{2}} = \dfrac{mv^2}{r}$

$\dfrac{mv^2}{r} = mg \qquad v = \sqrt{rg}$

16.(15)



$\Rightarrow \qquad a_{\max} = \mu g$



$\Rightarrow \qquad F_{\max} = (M + m)\mu g \qquad \Rightarrow \qquad F_{\max} = 15N$

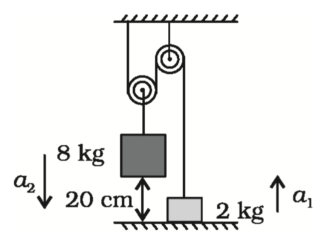
17. (A) $a_{2kg} = 2 \times a_{8kg}$; or $a_1 = 2a_2$

For 8 kg block;

$8g - 2T = 8 \times a_2$ ....(i)

For 2 kg block ;

$T - 2g = 2 \times a_1 = 2 \times (2a_2)$ ....(ii)



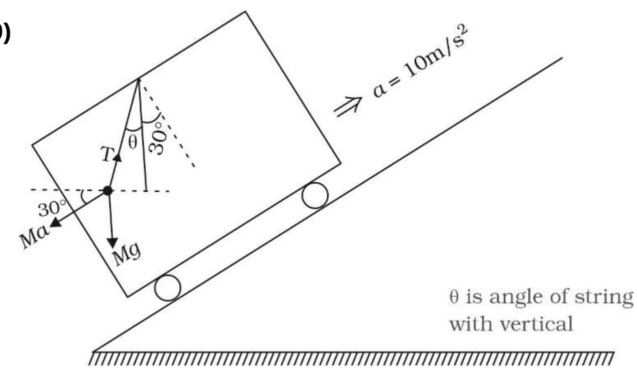
Solving (i) and (ii); $\qquad a_2 = 2.5m\ /\ s^2$

$\therefore \qquad$ For 8 kg block;

$s = ut + \dfrac{1}{2}a_2t^2$

$0.20 = 0 + \dfrac{1}{2} \times 2.5t^2 \qquad \Rightarrow \qquad t = 0.4\,\text{sec}$

18.(30)



w.r.t car bob will be in equilibrium

F.B.D of bob w.r.t car is given in above diagram

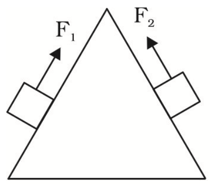
For equilibrium

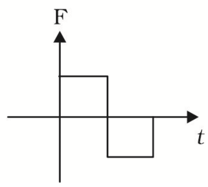
$T\cos\theta = Mg + Ma\sin 30°$ ...(i)

$T\sin\theta = Ma\cos 30°$ ...(ii)

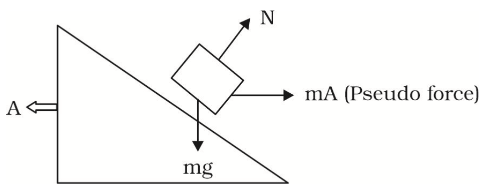
$\therefore \quad \tan\theta = \dfrac{a\cos 30°}{g + a\sin 30°} = \dfrac{1}{\sqrt{3}} \qquad \therefore \quad \theta = 30°$

19.(A) Direction of force will change but magnitude remain same





20.(B)



for wedge

$N\sin 30° = MA$ ... (1)

from the frame of wedge

$N = mg\cos\theta° - \dfrac{mA}{2}$ ... (2)

$mA\cos\theta + mg\sin\theta = ma_r$ ... (3)

from (1) (2) and (3) $\quad a_r = \dfrac{2}{3}g$

21.(3) Without friction, $t_1 = \sqrt{\dfrac{2l}{g\sin\theta}}$

with friction, $t_2 = \sqrt{\dfrac{2l}{g\sin\theta - \mu g\cos\theta}} = \alpha t_1$

so $\sqrt{\dfrac{2l}{g\sin\theta - \mu g\cos\theta}} = \alpha\sqrt{\dfrac{2l}{g\sin\theta}}$

$g\sin\theta = \alpha^2(g\sin\theta - \mu g\cos\theta) \Rightarrow \mu = \dfrac{\left(\alpha^2 - 1\right)g\sin\theta}{\alpha^2 g\cos\theta}$

comparing $x = 3$

22.(A) $F = F_0\left(1 - \left(\dfrac{t-T}{T}\right)^2\right)$

$a = \dfrac{F_0}{m}\left(1 - \dfrac{1}{T^2}(t^2 + T^2 - 2tT)\right) \quad ; \quad \int_0^v dv = \int_0^t \dfrac{F_0}{m}\left(1 - \dfrac{1}{T^2}(t^2 + T^2 - 2tT)\right)dt$

$v(t) = \dfrac{F_0}{m}\left(t - \dfrac{1}{T^2}\left(\dfrac{t^3}{3} + T^2 t - t^2 T\right)\right)\Bigg|_0^t \quad ; \quad v(2T) = \dfrac{F_0}{m}\left(2T - \dfrac{1}{T^2}\left(\dfrac{8T^3}{3} + 2T^3 - 4T^3\right)\right) \quad ; \quad \dfrac{4F_0 T}{3M}$

23.(3) $2t_A = t_D$

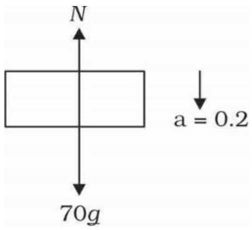
$\Rightarrow \quad 2\sqrt{\frac{25}{a_A}} = \sqrt{\frac{25}{a_D}} \quad \Rightarrow \quad 4a_D = a_A$

$\Rightarrow \quad 4g\left(\sin 30° - \mu \cos 30°\right) = g\left(\sin 30° + \mu \cos 30°\right)$

$\Rightarrow \quad 4 - 4\sqrt{3}\mu = 1 + \sqrt{3}\mu \quad \Rightarrow \quad 5\sqrt{3}\mu = 3$

$\Rightarrow \quad \mu = \frac{\sqrt{3}}{5} = \frac{\sqrt{x}}{5} \quad \therefore \quad x = 3$

24.(A)



$70g - N = 70 \times 6.2$

$700 - 14 = N$

$N = 686\,newtons$

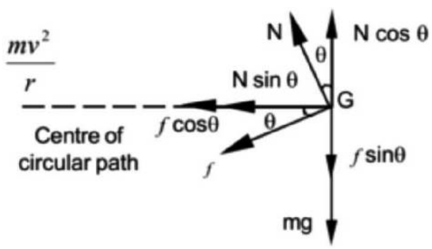
25.(D) $f = uN$

$N\cos 30° = f\sin 30° + mg$

$\Rightarrow \quad N \times 0.87 = 0.2 \times N \times 0.5 + \left(800 \times 9.8\right)$

$\Rightarrow \quad N\left(0.87 - 0.1\right) = 7840$

$\Rightarrow \quad N = 10181.81 \text{ Newton } = 10.2 \times 10^3 \text{ Newton}$



26.(A) $\vec{a} = \vec{F}/m = 8\hat{i} + 2\hat{j} \qquad \vec{u} = 0$

$\vec{s} = \frac{1}{2}\vec{a}\,t^2 = 400\hat{i} + 100\hat{j} = \vec{r}_f - \vec{r}_i$

Assuming $\vec{r}_i = 0$ (Since not given)

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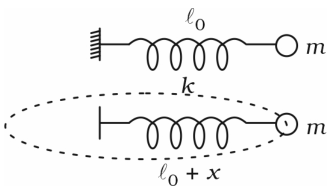
Dynamics of a Particle	Class - XI | Physics

JEE Main 2022

1.(C) Spring force $kx = m w^2 (\ell_0 + x)$



$\Rightarrow \quad x = \dfrac{m w^2 \ell_0}{k - m w^2}$

2.(B) $a = -\mu g = -0.5 \times 9.8 = -4.9 m / s^2$

$d = \dfrac{u^2}{2a} = \dfrac{9.8 \times 9.8}{2(4.9)} = 9.8m$

3.(C) $T = M\omega^2 R$

$T = 80N \quad M = 0.1 \quad \omega = ? \quad R = 2m$

$80 = 0.1\omega^2(2); \quad \omega^2 = 400$

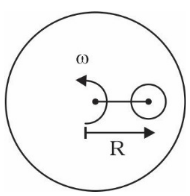
$\omega = 20; \qquad 2\pi f = 20$

$f = \dfrac{10}{\pi}\dfrac{rev}{s} = \dfrac{600}{\pi}\dfrac{rev}{\min}$

4.(2) $\mu(6 - l)\lambda g = \lambda \ell g$

$0.5(6 - l) = l \Rightarrow l = 2m$

5.(B) To rotate without slipping



$f_{\max} \geq m\omega^2 R$

$\mu mg \geq m\omega^2 R \quad \boxed{\therefore R \leq \dfrac{\mu g}{\omega^2}}$

6.(24) $\dfrac{V_{\max}^2}{R} = \mu g \Rightarrow \dfrac{(30)^2}{75} = \dfrac{V^2}{48}; \qquad V = \sqrt{\dfrac{(30)^2 \times 48}{75}} = 24 \text{ m/sec}$

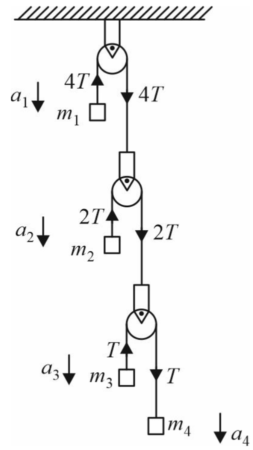
7.(B) Weight loss

Going down with acceleration

Then $\quad mg - N = ma$

$N = m(g - a)$

8.(A)

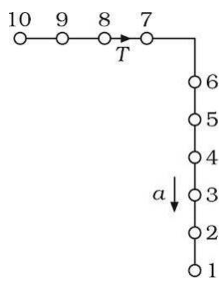


Using constraints ; $\sum \vec{T}.\vec{a} = 0$

$-4Ta_1 - 2Ta_2 - Ta_3 - Ta_4 = 0$

$4a_1 + 2a_2 + a_3 + a_4 = 0$

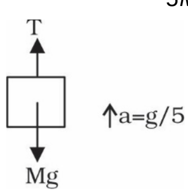
9.(36) $6\ mg = 10\ ma$



$\Rightarrow \quad a = \frac{3}{5}g$; $\quad T = 3\ ma = \frac{9}{5}mg = \frac{9}{5} \times 2 \times 10 = 36\ N$

10.(C) $F = \mu(M+m)g = 0.5(8+2)9.8 = 49N$

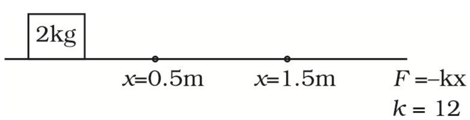
11.(6) $a_{common} = \frac{2Mg - Mg}{5M} = \frac{g}{5}$



For block M:

$T - Mg = M \times \frac{g}{5}$; $\quad T = \frac{6}{5}Mg$

12.(C)



$\frac{mvdv}{dx} = -kx$; $\quad m\int_4^v vdv = -k\int_{0.5}^{1.5} xdx$

$$\frac{v^2}{2} - \frac{16}{2} = -\frac{k}{m}\left[\frac{(1.5)^2}{2} - \frac{(0.5)^2}{2}\right]$$

$v^2 - 16 = \frac{-12}{2}(2.25 - 0.25)$; $\quad v = 2$

13.(C) $F = \rho A v^2 = v\dfrac{dm}{dt}\left[\dfrac{dm}{dt} = \rho A v\right]$

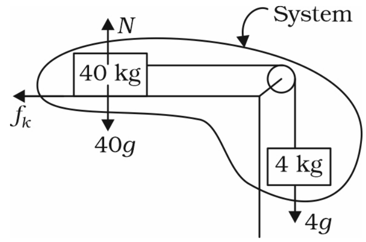
$= 1 \times 10$ Newton

$a = \dfrac{F}{m} = \dfrac{10}{2} = 5\dfrac{m}{s^2}$

14.(D) $N = 40g$

$f_k = \mu_k N = 0.02 \times 40g$

$f_k = 2 \times 4 = 8N$

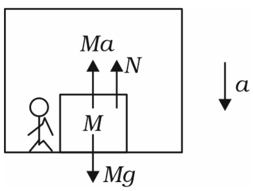


Along string on system

$4g - 8 = (4 + 40)a$

$\dfrac{32}{44} = a$; $\quad a = \dfrac{8}{11}$ ms^{-2}

15.(C) w.r.t lift



$Ma + N = Mg \quad \left(\because \; N = \dfrac{Mg}{4}\right)$

$Ma + \dfrac{Mg}{4} = Mg$

$a = g - \dfrac{g}{4}$; $\quad a = \dfrac{3g}{4}$

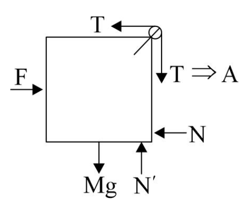
16.(B)

So change in momentum in each ball $= mv - (-mv) = 2mv$

So $F\Delta T = 2mv$; $\quad F = \dfrac{2mv}{\Delta T} = \dfrac{2 \times 0.05 \times 10}{0.005} = 200N$

17.(A) For M:

$F - T - N = MA$ ... (1)

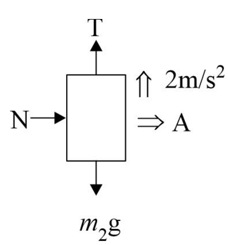


For m_2:

$T - m_2 g = m_2(2)$

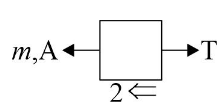
$T = m_2(g + 2) = 240$

$N = m_2 A$ ... (2)



For m_1 (w.r.t. M)

$m_1 A - T = m_1(2)$

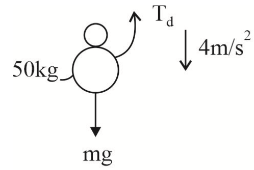


$10A - 240 = 20$; $\quad A = \dfrac{260}{10} = 26m/s^2$

Putting in (2) : $N = 20(26) = 520$

Putting in (1) : $F = T + N + MA = 240 + 520 + 2600 = 3360$

18.(C) $500 - T_d = (50)(4)$



$T_d = 300N$

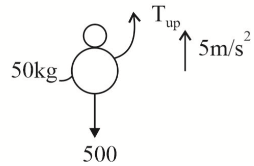
$T_d < 350N \Rightarrow$ does not break

$T_{up} - 500 = (50)(5)$

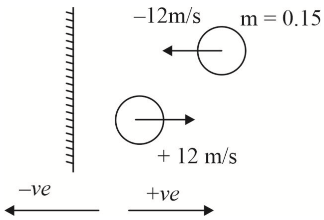
$T_{up} = 500 + 250$

$T_{up} = 750N$

$T_{up} > 350N \Rightarrow$ Breaks



19.(B)



$p_i =$ initial momentum before collision $= mu = -0.15 \times 12$

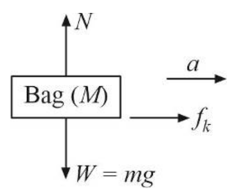
$p_f =$ final momentum after collision $= mV = +0.15 \times 12$

$\text{Force} = \dfrac{\text{change in momentum}}{\text{time}}$; $\text{Time} = \dfrac{\text{change in momentum}}{\text{Force}}$

$t = \dfrac{p_f - p_i}{F} = \dfrac{0.15 \times 12 - (-0.15 \times 12)}{100}$; $t = \dfrac{0.3 \times 12}{100} = \dfrac{3.6}{100} = 0.036\,s$

20.(B) $a = \left(\dfrac{M_2 - M_1}{M_1 + M_2}\right) g$; $a_1 = \dfrac{g}{3}$; $a_2 = \dfrac{2g}{4} = \dfrac{g}{2}$

21.(B) $N = mg$ and $f_k = ma$



$\Rightarrow \quad \mu N = ma$

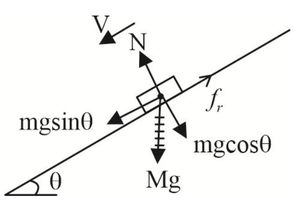
$\Rightarrow \quad \mu mg = ma$

$\Rightarrow \quad a = \mu g$

For uniformly – accelerate motion

$v^2 = u^2 + 2as \quad \Rightarrow \quad 2^2 = 0^2 + 2\mu gs \quad \Rightarrow \quad 4 = 2 \times 0.4 \times 10 \times s \quad \Rightarrow \quad s = \dfrac{1}{2} = 0.5\,m$

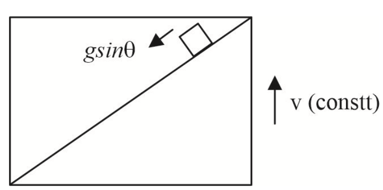
22.(A)



$\therefore \quad a = 0$; $\quad f_r = Mg\sin\theta$

$\therefore \quad$ Contact force $= \sqrt{N^2 + f_r^2} = \sqrt{(Mg\cos\theta)^2 + (Mg\sin\theta)^2} = Mg$

23.(C) $l = \dfrac{1}{2} \times (g \sin 30°) \times t^2 = \dfrac{1}{2} \times 10 \times \dfrac{1}{2} \times 4$



or $\quad l = 10\,m$

When $\quad \theta = 45°$

$l = ut + \dfrac{1}{2}(g \sin 45°) \times t_2^2$

or, $\quad 10 = 0 + \dfrac{1}{2} \times 10 \times \dfrac{1}{\sqrt{2}} \times t_2^2 \quad \Rightarrow \quad t_2 = \left(2\sqrt{2}\right)^{1/2} = 1.68\,\text{sec}$

24.(B) $F = \left|v_{rel}\left(\frac{dm}{dt}\right)\right|$

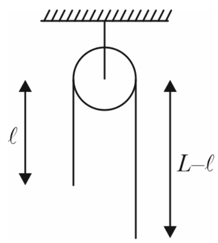
$F = (4.5\ cm\ /\ s) \times \left[\frac{10g}{5\sec}\right]$; $\quad F = 9dyne$

25.(D) Thrust applied by water on vertical wall

$F = \frac{dp}{dt} = \rho Av(v-0)$, $\quad F = \rho Av^2$

$= 1000 \times \frac{10}{10^4} \times (20)^2 \quad = 400\,N$

26.(D) $\lambda = \frac{\text{mass}}{\text{Length}}$



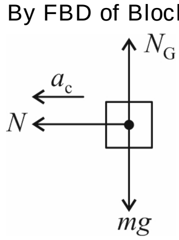
$\lambda(L-\ell)g - \lambda\ell g = \lambda La \quad$ . . . (i)

Given $\ell = \frac{L}{x}$, $a = \frac{g}{2}$

$\lambda g[L-2\ell] = \lambda La$

$g\left[L - \frac{2L}{x}\right] = L \times \frac{g}{2}$; $\quad \frac{L}{2} = \frac{2L}{x} \Rightarrow x = 4$

27.(A) By FBD of Block



$\Rightarrow \quad N = m.a_c$; $\quad N = \frac{mv^2}{R} = N \propto V^2$

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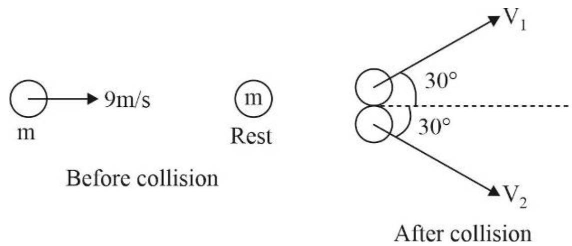
1.(1)



Conserving linear momentum in y-direction we have,

$mV_1 \sin 30° = mV_2 \sin 30°$

$\therefore \quad \frac{V_1}{V_2} = 1 \quad \therefore \quad x = 1$

2.(C) The given body can be seen as a superposition of :

(i) a disc of radius a centred at $x = a$ and mass m

(ii) a disc of radius $\frac{a}{2}$ centred at $x = \frac{3a}{2}$ and mass $\frac{-m}{4}$

So, $X_{CM} = \dfrac{m(a) + \left(\frac{-m}{4}\right)\left(\frac{3a}{2}\right)}{m + \left(\frac{-m}{4}\right)} = \frac{5a}{6}$

3.(2) $m_1 = 1$ kg ; $\quad m_2 = 2\ kg$

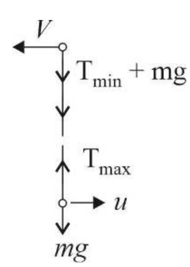
$m_1 v_2 = m_2 v_2 \Rightarrow \frac{v_1}{v_2} = 2$

Therefore, $\frac{k_1}{k_2} = \frac{m_1 v_1^2}{m_2 v_2^2} = \left(\frac{1}{2}\right)(2)^2 = 2$

4.(1) $F = -\frac{dU}{dr} = 0 \Rightarrow \frac{-10\alpha}{r^{11}} + \frac{5\beta}{r^6} = 0$

$\Rightarrow \quad \frac{10\alpha}{r^5} = 5\beta \quad \therefore \quad r = \left(\frac{2\alpha}{\beta}\right)^{1/5} \quad \therefore \quad a = 1,\ b = 5$

5.(5) $T_{max} - mg = \frac{mu^2}{R}; \quad T_{min} + mg = \frac{mV^2}{R}$



$T_{max} = 5T_{min}$

$\Rightarrow \quad mg + \frac{mu^2}{R} = 5\left(\frac{mV^2}{R} - mg\right)$

$\Rightarrow \quad 5V^2 = u^2 + 6gR \quad \ldots (1)$

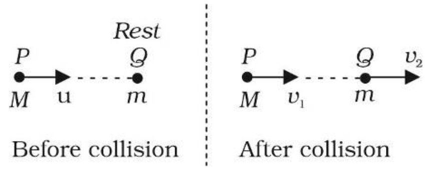
$\frac{1}{2}mV^2 = \frac{1}{2}mu^2 - 2mgR$

$V^2 = u^2 - 4gR$ ... (2)

By (1) and (2), $4V^2 = 10gR \Rightarrow \sqrt{\frac{5 \times 10 \times 1}{2}} = V$

6.(1) $p = \sqrt{2mKE}$; $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{16}} = \frac{1}{2}$; $n = 1$

7.(A)



From conservation of momentum;

$M \times u + m \times 0 = Mv_1 + mv_2$ ... (i)

$e = \frac{v_2 - v_1}{u - 0}$

or, $1 = \frac{v_2 - v_1}{u} \Rightarrow v_2 - v_1 = u$ ... (ii)

From (i) and (ii),

$M \times u = M \times (v_2 - u) + m \cdot v_2 \Rightarrow v_2 = \frac{2Mu}{(M + m)}$

Given $m << M$ $\therefore$ Maximum value of $v_2 = 2u$

$\Rightarrow$ Assertion (A) is correct and Reason (R) is the correct explanation of A.

8.(C) $m = 0.5kg, u = 20ms^{-1}$ $\because K_f = 0.05K_i$

$\Rightarrow \frac{1}{2}mv^2 = 0.05\frac{1}{2}mu^2 \Rightarrow v = \frac{\sqrt{5}}{10}u$

$\Rightarrow v = \frac{2.23}{10} \times 20m/s$ $\therefore$ $v = 4.47m/s$

9.(30) Conserving linear momentum along x axis

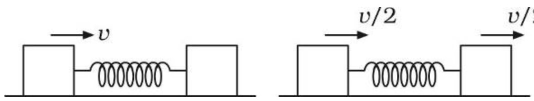
$10 \times 10\sqrt{3} = 10 \times 20\cos\theta$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30°$

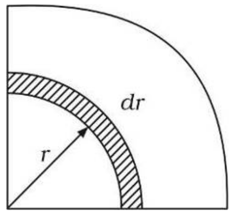
10.(A) For maximum compression, collision between A and C is elastic.

$v_A = v$



$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{2k}}$

11.(4) *gf* a is the radius $dA = \sigma \frac{\pi r}{2} . dr$



$$x_{cm} = \frac{1}{\left(\frac{\pi}{4}a^2\right)} \int_0^a \left(\frac{2r}{\pi}\right) dA = \frac{4a}{3\pi}$$

12.(D) $P = Fv$; $P = mav$

$$P\int dt = m\int v\, dv$$

$$m\frac{v^2}{2} = Pt \;; \quad v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\frac{dx}{dt} = \left(\frac{2Pt}{m}\right)^{1/2} \;; \; x = \left(\frac{2P}{m}\right)^{1/2} \frac{t^{3/2}}{3/2}$$

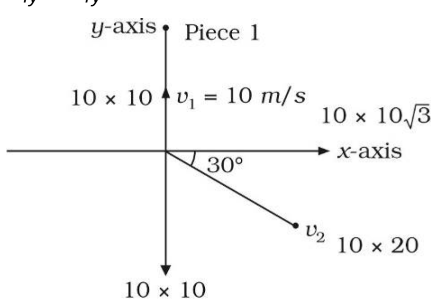
$$x \propto t^{3/2}$$

13.(10) $mg\Delta h = \frac{1}{2}mv^2$

$$2 \times 10 \times 5 = v^2 \Rightarrow v = 10\ m/s$$

14.(20) $P_{ix} = P_{fx} = 10\sqrt{3} \times 10$

$$P_{iy} = P_{fy} = 10 \times 10 - 10 \times 10$$



$$v_2 = 20$$

15.(10) $F = \left(\frac{v^2 - u^2}{2s}\right)m = -\frac{100 \times 0.1}{2 \times 1/2}$

$= -10\ N$

16.(A) $U(r) = \frac{-C}{r}$

$$F = \frac{-dU}{dr} = -C \times \frac{1}{r^2}$$

$$\frac{mV^2}{r} = \frac{C}{r^2}$$

$$V^2 \propto \frac{1}{r}$$

17.(A) Applying conservation of linear momentum

$M_1U = -M_1V + M_2V$; $M_1U = (M_2 - M_1)V$

$e = 1 = \frac{2V}{U}$; $2V = U$

$2M_1V = (M_2 - M_1)V$; $3M_1 = M_2$

$M_2 : M_1 = 3 : 1$

18.(6) From energy conservation,

$\frac{1}{2}mV^2 = \frac{1}{2}kx^2$ $\Rightarrow \frac{1}{2}(4)(10)^2 = \frac{1}{2}(100)x^2$ $\therefore x = 2m$

$\therefore$ Length of compressed spring $= 8 - 2 = 6m$

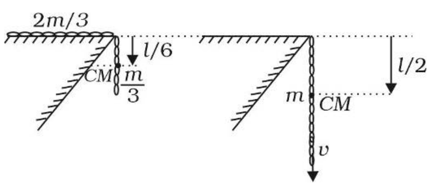
19.(B) Let velocity of bullet before collision = u m/s

$\Rightarrow$ conservation of linear momentum for collision: $(0.01)u = 6v$

Where $v =$ velocity of (block + bullet) just after collision.

Now, $v = \sqrt{2.gh} = \sqrt{2 \times 9.8 \times \frac{9.8}{100}}$; $u = \frac{600 \times 9.8}{\sqrt{50}} m/s = 831.5 m/s$

20.(40) $\frac{1}{2}mV^2 = \frac{mgl}{2} - \frac{m}{3}.g.\frac{l}{6}$



$\frac{1}{2}mV^2 = \frac{8}{18}mgl = 40J$

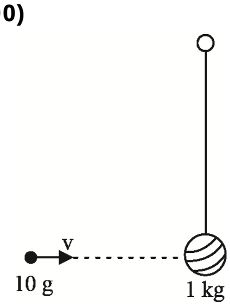
21.(2) $W_A = W_B$

$F_A d \cos 45 = F_B d \cos 60$

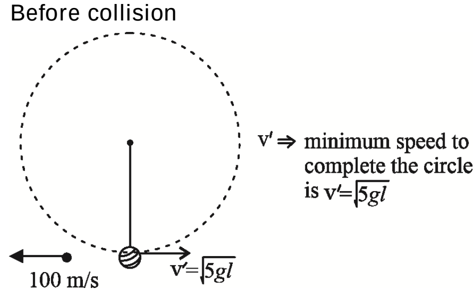
$F_A \cos 45 = F_B \cos 60$

$\frac{F_A}{F_B} = \frac{1}{\sqrt{2}}$; So $x = 2$

22.(400)



Before collision



After collision $v' = \sqrt{5 \times 10 \times \frac{1}{2}} = 5 m/s$

By conservation of linear momentum

$$(P_i)_{system} = (P_F)_{system}$$

$$m_b v + 0 = m_{bob} v' + m_b v_b$$

$$10 \times 10^{-3} v = 1 \times 5 + 10 \times 10^{-3} \times (-100)$$

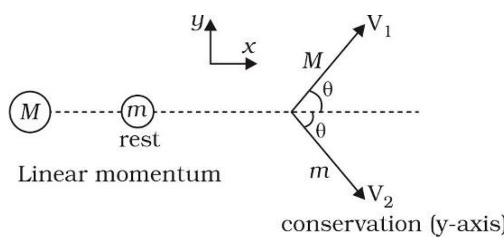
$$10 \times 10^{-3} v = 5 - 1000 \times 10^{-3}$$

$$10 \times 10^{-3} v = 4$$

$$v = \frac{4}{10 \times 10^{-3}} = 400 m/s$$

23.(B) $MV_1 \sin\theta = mV_2 \sin\theta$

$MV_1 = mV_2$ ... (i)



In x direction

$MV_0 = mV_2 \cos\theta + MV_1 \cos\theta$

$MV_0 = mV_2 \cos\theta + mV_2 \cos\theta$ (from (i))

$MV_0 = 2mV_2 \cos\theta$ ... (ii)

$e = 1$ given

$$1 = \frac{V_2 - V_1 \cos 2\theta}{V_0 \cos\theta}$$

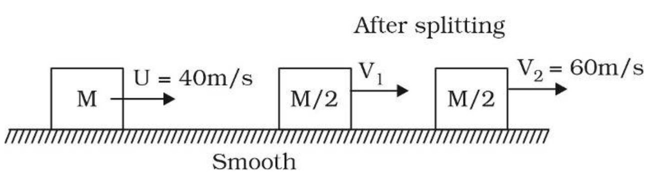
$V_2 - V_1 \cos 2\theta = V_0 \cos\theta$; $\quad V_2 - \frac{mV_2}{M} \cos 2\theta = V_0 \cos\theta$

$V_2 \left[1 - \frac{m}{M} \cos 2\theta\right] = V_0 \cos\theta$; $\quad \frac{MV_0}{2m \cos\theta} \left[1 - \frac{m}{M} \cos 2\theta\right] = V_0 \cos\theta$

$M - m \cos 2\theta = 2m \cos^2\theta$; $\quad \frac{M}{m} = (2\cos^2\theta + \cos 2\theta) = 4\cos^2\theta - 1$

$$\left(\frac{M}{m}\right)_{max} = 4 - 1 = 3$$

24.(1)



Applying conservation of momentum in horizontal direction

$$mU = \frac{mV_1}{2} + \frac{mV_2}{2}$$

$\Rightarrow V_1 = 2U - V_2 \qquad \Rightarrow V_1 = 20 m/s$

Now,

$$(K.E.)_{initial} = \frac{1}{2} mV^2 = 800m$$

$$(K.E.)_{final} = \frac{1}{2}\left(\frac{m}{2}\right)V_1^2 + \frac{1}{2}\left(\frac{m}{2}\right)V_2^2 = 1000m$$

$$\Delta KE = 200m$$

$$\therefore \frac{\Delta KE}{K.E_{initial}} = \frac{200m}{800m} = \frac{1}{4} = \frac{x}{4} \quad \Rightarrow x = 1$$

25.(A) Conservation of linear momentum $(3m)\times 40=(2m)v+(m)\times 60$

$v=30m\ /\ s$

$k_i=\frac{1}{2}\times(3m)\times(40)^2=\frac{m}{2}\times 4800$

$k_f=\frac{1}{2}\times(2m)\times(30)^2+\frac{1}{2}\times(m)\times(60)^2=\frac{m}{2}\times 5400$

fractional charge = $\frac{k_f-k_i}{k_i}=\frac{\frac{m}{2}\times 5400-\frac{m}{2}\times 4800}{\frac{m}{2}\times 4800}=\frac{1}{8}$

26.(C) $W_{mg}=W_f=\Delta k$

$mgh+W_f=\frac{1}{2}m\times 0.8^2\times gh$

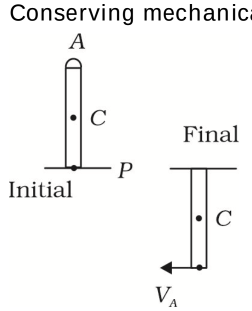
$W_f=-0.68mgh$

27.(16) Only 10% of kE got converted to spring potential energy

$\frac{1}{2}mv^2\times\frac{10}{100}=\frac{1}{2}kx^2$, solving $k=16\times 10^5$

Here $V-20m\ /\ s,\ m=4\times 10^4 kg$

28.(6) Conserving mechanical energy



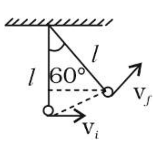
$mgl=\frac{1}{2}I_P w^2$

$I_P=\frac{ml^2}{3},\ w=\sqrt{\frac{6g}{l}}$

$V_A=wl=\sqrt{6gl}$

$V_A=6$

29.(2)



Loss in K.E. = Gain to P.E.

$\Rightarrow\frac{1}{2}\times mv_i^2-\frac{1}{2}\times mv_f^2=mg\left(l-l\cos 60^\circ\right)$

$\Rightarrow V_f=\sqrt{V_i^2-2gl\left(1-\frac{1}{2}\right)}=\sqrt{(3)^2-2\times 10\times 0.5\times\frac{1}{2}}=2m\ /\ s$

30.(450) $F=(5y+20)\hat{j}$

$$\omega=\int Fdy=\int_0^{10}(5y+20)\,dy=\left(\frac{5y^2}{2}+20y\right)_0^{10}=\frac{5}{2}\times 100+20\times 10=250+200=450\ J$$

31.(C) u = 0; P = F.V

P = F.V; P = mav

$\frac{P}{m}=\frac{vdv}{dx}$; $\int_0^v v^2 dv=\int_0^x \frac{P}{m}dx$

$\frac{v^3}{3}=\frac{P}{m}x$; $v=\left(\frac{3Px}{m}\right)^{1/3}$

$\frac{dx}{dt}=\left(\frac{3P}{m}\right)^{1/3} x^{1/3}$; $\int_0^x \frac{dx}{x^{1/3}}=\left(\frac{3P}{m}\right)^{1/3}\int_0^t dt$

$\frac{x^{2/3}}{2/3}=\left(\frac{3P}{m}\right)^{1/3} t$; $x^{2/3}=\frac{2}{3}\left(\frac{3P}{m}\right)^{1/3} t$

$x^2=\left(\frac{8}{9}\frac{P}{m}\right)t^3$; $x=\left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$

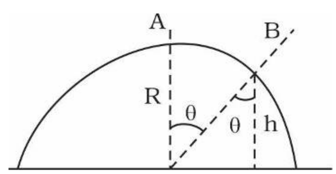
32.(A) At $x<x_1$ PE = 8J

Mechanical Energy = 8J

$\therefore$ $KE=0$ $\therefore$ $V=0$

33.(2) at critical point $mg\cos\theta=\frac{mv^2}{R}$

$Rg\cos\theta=v^2$



Conserving energy

$\frac{1}{2}mv^2=mg\,(R-h)$

$\frac{1}{2}m\,(Rg\cos\theta)=mg\,(R-R\cos\theta)$

$\cos\theta=\frac{2}{3}$ $\therefore$ $h=R\cos\theta=\frac{3\times 2}{3}=2\text{m}$

JEE Advanced 2021

1.(0.5) $H=\frac{(5\sqrt{2})^2\sin^2 4s}{2g}=\frac{50\times 1}{20\times 2}=\frac{50}{40}=\frac{5}{4}m$

$T=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times 5/4}{10}}=\frac{1}{\sqrt{4}}=0.5\text{ sec}$

Ans. 0.5

2.(7.5) Using position of COM

$r_{CM}=\frac{m_1r_1+m_2r_2}{m_1+m_2}$ $\Rightarrow$ $R=\frac{\frac{m}{2}\times\frac{R}{2}+\frac{m}{2}\times x}{m}$

$R=\frac{R}{4}+\frac{x}{2}$; $\frac{3R}{4}=\frac{x}{2}$

$x=\frac{3R}{2}=\frac{3}{2}\times\frac{u^2\sin 2\theta}{g}=\frac{3}{2}\times\frac{50\times 1}{10}=7.5$

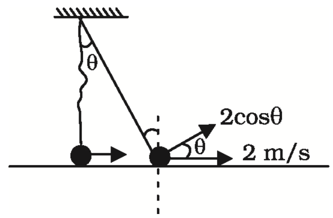
3.(0.18) **4.(0.16)**

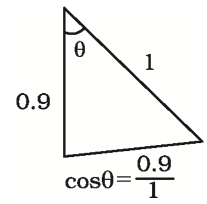
Angular momentum

$J = r_{\perp} p$

$= 0.9 \times 0.2$

$= 0.18 \text{ kg m}^2/\text{s}$





Velocity $\perp_r$ to the string, $V_1 = v\cos\theta$

$= 2 \times 0.9$

$= 1.8 \text{ m/s}$

KE just after lift off $= \frac{1}{2} \times 0.1 \times (1.8)^2$

$= 0.162$

$= 0.16 J$

Solutions of Archive - JEE Main & Advanced

Energy and Momentum	Class - XI | Physics

JEE Main 2022

1.(C) $\Delta K.E. = W$

$W = \int \vec{F}.(dx\hat{i} + dy\hat{j})$ (also, $\vec{F} = 4x\hat{i} + 3y^2\hat{j}$)

$= \int_1^2 4x\,dx + \int_2^3 3y^2 dy \qquad = (2x^2)_1^2 + (y^3)_2^3$

$= (8-2) + (27-8) \qquad = 6 + 19 = 25\,J$

2.(120) By work energy theorem,

$w_{gravity} + w_{spring} = \Delta K$

$mg\left(H + \frac{H}{2}\right) - \frac{1}{2}K\left(\frac{H}{2}\right)^2 = K_f - K_i$

$mg\frac{3H}{2} - \frac{1}{2}K\frac{H^2}{4} = 0 - 0$; $\qquad \frac{3mgH}{2} = \frac{KH^2}{8}$

$\frac{3(0.1)\times 10}{2} = \frac{K(0.1)}{8}$; $\qquad K = 120\,N/m$

3.(B) Since speed is uniform, the magnitude of centripetal acceleration will be constant

$\Rightarrow$ At topmost point, $T + 3g = \frac{mV^2}{R}$ = constant

$\Rightarrow T = \frac{mV^2}{R} - mg$ which is minimum

4.(C) $v = \frac{A}{r^{10}} - \frac{B}{r^5}$

$F = -\frac{du}{dr} = 0$ (at equilibrium)

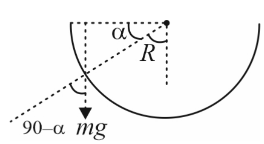
$\Rightarrow F = \frac{-10A}{r^{11}} + \frac{5B}{r^6} = 0 \quad \Rightarrow r^5 = \frac{2A}{B} \qquad \Rightarrow r = \left(\frac{2A}{B}\right)^{1/5}$

5.(600) $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 \qquad \Rightarrow \qquad k = 600\,N/m$

6.(A) At highest point velocity = 0

7.(C) $mg(R\sin\alpha) = \frac{1}{2}mv^2$

$v^2 = 2gR\sin\alpha$



Centripetal force $= \frac{mv^2}{R} = 2mg\sin\alpha$

$N - mg\sin\alpha = \frac{mv^2}{2}$; $\qquad N = mg\sin\alpha$

Ratio $A = \frac{2}{3}$

8.(A) Work done = Area under force-displacement curve
Area above x-axis = $+ve$ work done
Area below x-axis = $-ve$ work done

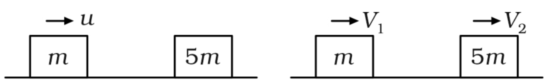
9.(12) $I = \Delta P = 2\,mv \quad = 2 \times 0.4 \times 15 = 12$

10.(C) As COM remains unchanged

$$m_1 x_1 + m_2 x_2 = 0 \qquad \Rightarrow \qquad 10 \times 6 + 30 x_2 = 0$$

$$\Rightarrow \qquad x_2 = \frac{-10 \times 6}{30} = -2cm\ ; \quad 2cm \text{ towards } 10kg \text{ block}$$

11.(C)



$mu + 5m \times 0 = mV_1 + 5mV_2$

$\Rightarrow \qquad 5V_2 + V_1 = u \qquad \ldots\text{(i)}$

Also, $\quad V_2 - V_1 = u \qquad \ldots\text{(ii)}$

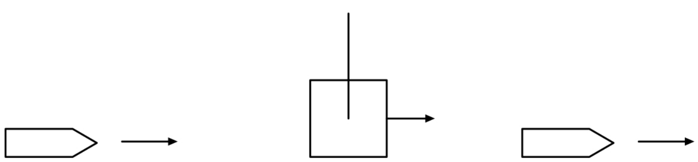
From (i) and (ii),

$6V_2 = 2u \Rightarrow V_2 = \dfrac{u}{3}\ ; \qquad KE \text{ of } m = \dfrac{1}{2}mu^2$

$KE \text{ of } 5m \text{ after collision } = \dfrac{1}{2}5m\left(\dfrac{u}{3}\right)^2 \quad = \dfrac{5}{18}mu^2$

Percentage transferred $= \dfrac{\frac{5}{18}mu^2}{\frac{1}{2}mu^2} \times 100 = 55.6\%$

12.(10) $v_2 = \sqrt{5gl} = \sqrt{5 \times 10 \times 2} = 10m\ /\ s$



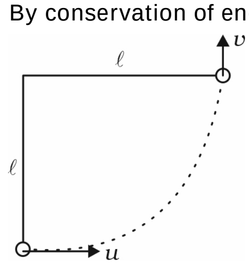
$u_1 = v \qquad\qquad v_2 \qquad\qquad v_1 = \dfrac{v}{3}$

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$75 \times v + 0 = 75\dfrac{u}{3} + 50 \times 10$

$50v = 500\ ; \qquad v = 10m\ /\ s$

13.(B) By conservation of energy

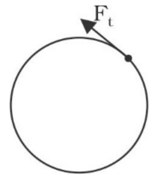


$v = \sqrt{u^2 - 2g}$

$\Rightarrow \qquad$ Change in velocity: $\overrightarrow{\Delta v} = v\hat{j} - u\hat{i}$

$\qquad |\overrightarrow{\Delta v}| = \sqrt{v^2 + u^2} = \sqrt{2\left(u^2 - 2g\ell\right)}\ ; \quad x = 2$

14.(A) $a_r = \frac{v^2}{r} = k^2 r t^2$

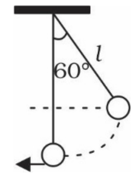


$v = krt \quad \therefore a_t = \frac{dv}{dt} = kr =$ constant

$\therefore \quad$ Power $= F_t . v = (m a_t) v$

$= (mkr)v = (mkr)(krt) = mk^2 r^2 t$

15.(5) Max. speed will be at lowest from conservation of energy:



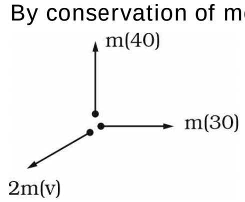
$mg \times \left(l - \frac{l}{2} \right) = \frac{1}{2} m v_{max}^2$

$\sqrt{2gl \times \frac{l}{2}} = V_{max} \quad \Rightarrow V_{max} = \sqrt{10 \times 2.5} = 5 m\ /\ s$

16.(6) $60 \times v = (120 + 60) \times 2$; $\quad v = \frac{180 \times 2}{60} = 6$ m / s

17.(B) Say masses are $m, m, 2m$

By conservation of momentum :

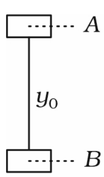


$(2m)v = \sqrt{(30m)^2 + (40m)^2}$; $\qquad v = 25 m\ /\ s$

18.(D) $m = \frac{1}{2} kg$; $\qquad W = \Delta K = \frac{1}{2} \times \frac{1}{2} \times \left(v_4^2 - v_0^2 \right)$

$= \frac{1}{2} \times \frac{1}{2} \times \left(\frac{1}{4} \times 4^{5/2} \right)^2 \qquad = \frac{1}{4} \times 64 = 16 J$

19.(D)



$U_1 = mgy_0$
$k_1 = 0$

$U_2 = 0$
$k_2 = ?$

By energy conservation

$U_1 + k_1 = U_2 + k_2$; $\qquad mgy_0 + 0 = 0 + k_2$; $\qquad k_2 = mgy_0$

20.(2) $\frac{1}{2} m u^2 = mgh$

$u = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2}$ m / s

$\therefore \quad v_{f,B} = u - g \sin 45° \times t_1$

$0 = 10\sqrt{2} - 10 \times \frac{1}{\sqrt{2}} t_1$; $\quad t_1 = 2$ sec

For Path BC:

$S_{BC} = ut + \frac{1}{2} (g \sin 30°) t_2^2$

$\frac{10}{\sin 30°} = 0 + \frac{1}{2} \times 10 \times \frac{1}{2} \times t_2^2$; $\qquad t_2 = \sqrt{\frac{20 \times 4}{10}} = 2\sqrt{2}$ sec

Total time $= t_1 + t_2 = 2(\sqrt{2} + 1)$ sec

21.(B) Initial velocity $= 4\,m/s$ at $x = 0$

Final velocity $= 16\,m/s$ at $x = 2$

Work done $= \Delta K.E.$

$$= \frac{1}{2}m\left(V_f^2 - V_i^2\right) \qquad = \frac{1}{2}\times\frac{1}{2}(16^2 - 4^2)$$

$$= \frac{1}{4}\times(256-16) = \frac{240}{4} = 60\,J$$

22.(B) Conservation of momentum

$mu + 0 = V(M+m)$

$0.2\times 10 = V(9.8+0.2)$

$2 = V(10)$; $\quad V = 0.2\,m\,/\,\text{sec}$

$\Delta K = \frac{1}{2}mu^2 - \frac{1}{2}(M+m)V^2$

$= \frac{1}{2}\times 0.2\times 100 - \frac{1}{2}\times 10\times 0.04 \qquad = 10 - 0.2 = 9.8\,J$

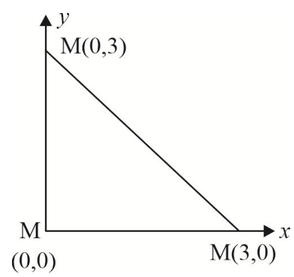
23.(2) $x_{cm} = \dfrac{M\times 0 + 3\times M + M\times 0}{3M} = 1m$

$y_{cm} = \dfrac{M\times 0 + M\times 0 + M\times 3}{3M} = 1m$

$\vec{r}_{cm} = 1\hat{i} + 1\hat{j}$

$\vec{r}_{cm} = \sqrt{2}$

$x = 2$



24.(B) Method (1)

$\because \quad \frac{1}{2}Kx_{\max}^2 = \frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(v_1 - v_2)^2 \Rightarrow \quad x_{\max} = \sqrt{\frac{m_1m_2}{(m_1+m_2)K}}.(v_1 - v_2)$

$\Rightarrow \quad x_{\max} = \sqrt{\frac{0.25\times 0.25}{0.50\times 2}}(2v) \qquad \therefore \quad x_{\max} = \frac{v}{2}$

Method (2)

Momentum conservation : $m(v) + m(-v) = 2mv_f$; $\qquad v_f = 0$

Energy conservation : $\frac{1}{2}mv^2 \times 2 = 0 + \frac{1}{2}kx_{\max}^2$

$\Rightarrow \quad x_{\max} = \sqrt{\frac{2m}{k}}.v \qquad \Rightarrow \quad x_{\max} = \sqrt{\frac{2\times 0.25}{2}}v \qquad \therefore \quad x_{\max} = \frac{v}{2}$

25.(B) k = kinetic energy $k = \dfrac{p^2}{2m}$

p = momentum

let p_1 be momentum of First body

p_2 be momentum of second body

$\dfrac{p_1^2}{2m_1} = \dfrac{p_2^2}{2m_2} \Rightarrow$ as kinetic energy is same

$\dfrac{p_1^2}{p_2^2} = \dfrac{m_1}{m_2}$; $\qquad m_1 = 8kg;\ m_2 = 2kg$

$\dfrac{p_1}{p_2} = \sqrt{\dfrac{m_1}{m_2}} = \sqrt{\dfrac{8}{2}} = \dfrac{2}{1}$

26.(D) $F = \frac{dm}{dt} v_{rel} = 0.5 \times 5 = 2.5N$

$P = Fv = 2.5 \times 5 = 12.5\ W$

27.(B) Impulse in both cases $= |\Delta P| = 5 \times 25 = 125\,kg\ m/s$

While force in first case $= \frac{125}{3} N$ and force in second case $= \frac{125}{5} = 25N$

28.(24) Conserving energy

$E = \frac{1}{2}(nE)\left(\frac{1}{4}\right)^2 + \frac{E}{4}$; $\frac{3E}{4} = \frac{nE}{32} \Rightarrow n = 24$

29.(A) $\frac{1}{2}mv^2 = 40J$; $v^2 = 400$

$v = 20$ ... (i)

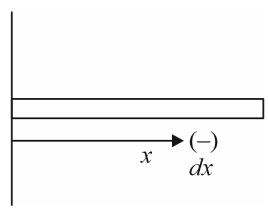
also $\frac{1}{2}mu^2 = 90J$, $u^2 = 900$, $u = 30$ ... (ii)

here, $v = u + at$, $20 = 30 + a(1) \Rightarrow a = 10\,m/s^2$ ... (iii)

When it stops $v_f = 0$

Now, $v_f^2 = u^2 + 2as$; $0 = (30)^2 - 2 \times 10 \times 5$; $s = 45$

30.(8) $dm = \rho dx = \rho_0 \left(1 - \frac{x^2}{L^2}\right) dx$



$$X_{com} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L x \rho_0 \left(1 - \frac{x^2}{L^2}\right) dx}{\int_0^L \rho_0 \left(1 - \frac{x^2}{L^2}\right) dx}$$

$x_{com} = \frac{3L}{8} = \frac{3L}{d} \Rightarrow d = 8$

31.(A) $\vec{r}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \frac{1(\hat{i} + 2\hat{j} + \hat{k}) + 3(-3\hat{i} - 2\hat{j} + \hat{k})}{1 + 3}$

$= -2\hat{i} - \hat{j} + k$; $|\vec{r}_{CM}| = \sqrt{6}$ same as of (A)

32.(C) $P_i = P, P_f = P + \frac{P(20)}{(100)} = 1.2P$

$\boxed{K = \frac{p^2}{2m}}$ %Change $= \left(\frac{K_f - K_i}{K_i}\right) \times 100 = (1.44 - 1) \times 100 = 44\%$

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Solutions of Archive - JEE Main & Advanced

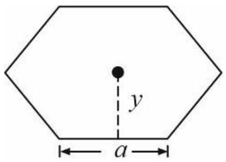
Rotational Motion	Class - XI | Physics

JEE Main 2021

1.(A) $I_1 = \dfrac{MR^2}{2} \qquad I_2 = \dfrac{MR^2}{2}$

$I_3 = \dfrac{MR^2}{2} \qquad I_4 = \dfrac{2}{5}MR^2 \qquad \Rightarrow \qquad I_1 = I_2 = I_3 > I_4$

2.(8)



$y = \dfrac{\sqrt{3}}{2}a$

Let the length and mass of each side be a *and* m.

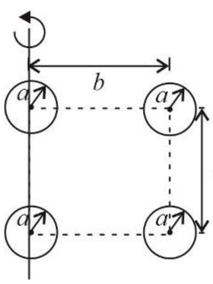
Then, $a = \dfrac{2.4}{6} = 0.4$ m ; $\qquad m = \dfrac{6}{6} = 1$ kg

Moment of inertia of one side, $I_1 = \dfrac{1}{12}ma^2 + m\left(\dfrac{\sqrt{3}}{2}a\right)^2 = \dfrac{5}{6}ma^2$

So, total moment of inertia, $I_{\text{Total}} = 6I_1 = 5\,ma^2$

$\Rightarrow \qquad I_{\text{Total}} = 5(1)\,(0.4)^2 = 0.8$ kg m^2

3.(D)



$$I = \left\{\frac{2}{5}ma^2 + \frac{2}{5}ma^2\right\} + \left(\frac{2}{5}ma^2 + mb^2\right) + \left(\frac{2}{5}ma^2 + mb^2\right) = \frac{8}{5}ma^2 + 2mb^2$$

4.(A) $mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$; $\qquad \dfrac{2\left(mgh - \dfrac{1}{2}mv^2\right)}{I} = \omega^2$

$\dfrac{2mgh - mv^2}{I} = \omega^2$; $\qquad \dfrac{2mgh}{I} - \dfrac{m\omega^2 r^2}{I} = \omega^2$

$v = r\omega$; $\qquad \dfrac{2mgh}{I} = \omega^2\left(\dfrac{(mr^2)}{I} + 1\right)$; $\qquad \omega^2 = \dfrac{2mgh}{I\left(\dfrac{mr^2}{I} + 1\right)}$

5.(Bonus) $\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2 = mgh$

Where h is the maximum height.

$\frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\frac{v_0^2}{a^2} = mgh$; $\quad h = \frac{7}{10}\frac{v_0^2}{g}$

also $\quad \frac{h}{x} = \sin\theta$ (where x is the maximum distance along the inclined plane.)

$x = \frac{h}{\sin\theta}$; $\quad \frac{7v_0^2}{10\, g\sin\theta}$

6.(C) $\vec{\tau}_0 (w.r.t.O) = \overrightarrow{OP} \times \vec{F} = \left(5\hat{i} + 5\sqrt{3}\,\hat{j}\right) \times \left(4\hat{i} - 3\hat{j}\right) = -20\sqrt{3}\,\hat{k} - 15\hat{k}$

$\tau_0 = -20\sqrt{3} - 15$

$\vec{\tau}_Q (w,r,t,Q) = \overrightarrow{QP} \times \vec{F} = \left(-5\hat{i} + 5\sqrt{3}\,\hat{j}\right) \times \left(4\hat{i} - 3\hat{j}\right) = \left(-20\sqrt{3} + 15\right)\hat{k}$

$\tau_Q = -20\sqrt{3} + 15$

7.(D) $\vec{L}_A = I_A\vec{\omega} = I_A\omega\hat{k}$

$\vec{L}_A = Mr^2\omega\hat{k} =$ constant vector

$\vec{L}_B = M\overrightarrow{BM} \times \vec{v}$

$L_B = M\left|\overrightarrow{BM}\right| v\sin 90°$

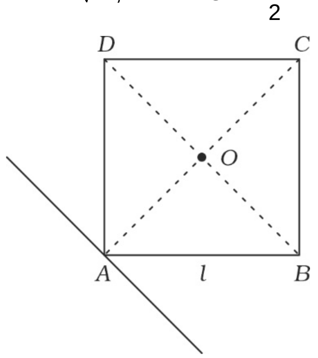
$L_B = Mvl =$ constant in magnitude but varies in direction.

8.(4) $\frac{1}{2}at^2 = S$; $\quad t = \sqrt{\frac{2S}{a}}$

$a = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}} \Rightarrow a_{max}$ for solid sphere

$t_{min} = \sqrt{\frac{2S}{a_{max}}} \Rightarrow$ solid sphere

9.(A) $AC = l\sqrt{2}$; $\quad AO = \frac{l\sqrt{2}}{2} = \frac{l}{\sqrt{2}}$



Moment of Inertia about this axis

$I = I_A + I_B + I_D + I_C$

$= 0 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(l\sqrt{2}\right)^2$

$= \frac{ml^2}{2} + \frac{ml^2}{2} + 2ml^2 \quad = 3ml^2$

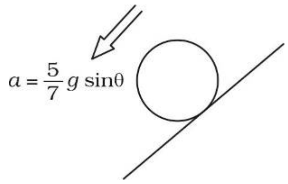
10.(20) $\tau = I\alpha$

$$20 \times 0.2 = \frac{20 \times 0.2^2}{2} \times \alpha$$

$$\alpha = 10 \ \text{rad/sec}^2$$

$$\& \ \omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \qquad \theta = \frac{50^2}{2 \times 10} = 125 \ \text{rad} \qquad \text{or} \qquad \theta = 2\pi n = 125 \ \text{rad}$$

$$n = \frac{125}{2\pi} = 19.9 \ \text{rev}$$

11.(D) 

$$a = \frac{5}{7} g \sin 30^\circ = \frac{5}{7} \times 9.8 \times \frac{1}{2} = 3.5 \ m/s^2$$

$$s = 0 \quad \Rightarrow \qquad t = \frac{24}{a} = \frac{2}{3.5} = 0.57 \ s$$

12.(D) Using angular momentum conservation

$$L_i = MR^2\omega$$

$$L_f = (MR^2 + 2mR^2)\omega'$$

$$\omega' = \left(\frac{M\omega}{M + 2m}\right)$$

13.(A) $L = \pi R$

$$R = \frac{L}{\pi}$$

$$I = MR^2 = \frac{ML^2}{\pi^2}$$

14.(C) For cylinder to be in equilibrium

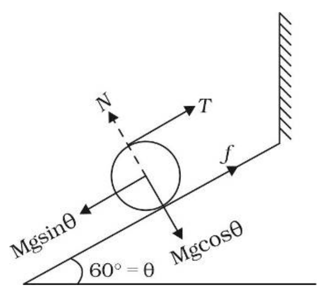
$$T \times R = f \times R$$

And $T + f = mg\sin\theta \Rightarrow f = \dfrac{mg\sin\theta}{2} = \dfrac{\sqrt{3}mg}{4}$

$$f_{\max} = \mu N = \mu Mg\cos\theta = 0.4 \times Mg\cos 60^\circ = \frac{Mg}{5}$$

As $f > f_{\max} \Rightarrow$ cylinder is slipping

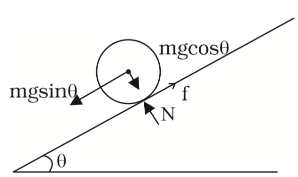
$$f = f_{\max} = \frac{mg}{5}$$



15.(3) Torque about bottom point, $mg\sin\theta . r = \dfrac{3}{2}mr^2\alpha$

$$\Rightarrow \qquad \alpha = \frac{2}{3}\frac{g\sin\theta}{r}$$

Acceleration of disc $= a = \alpha r = \dfrac{2}{3} g\sin\theta$



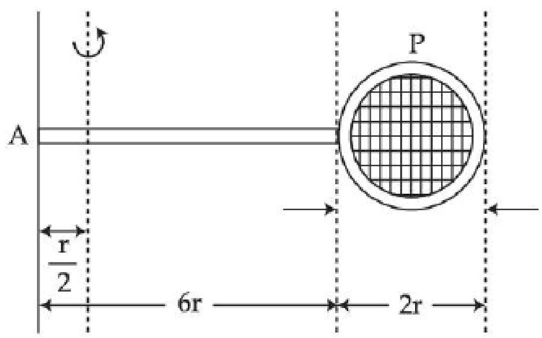
16.(20) $\vec{r} = \left(2\hat{i} + 0\hat{j} + 0\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = \left(-3\hat{j} - 4\hat{k}\right)$

$\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore \quad \vec{\tau} = \vec{r} \times \vec{F} - = \left(-3\hat{j} - 4\hat{k}\right) \times \left(4\hat{i} + 3\hat{j} + 4\hat{k}\right) = -16\hat{j} + 12\hat{k}$

$\therefore \quad \left|\vec{\tau}\right| = \sqrt{(16)^2 + (12)^2} = 20\,unit$

17.(52) $M.I.$ of Rod



$$I_1 = \frac{M(6r)^2}{12} + M\left(\frac{5r}{2}\right)^2 = \frac{37}{4}mr^2$$

$M.I.$ of ring

$$I_2 = \frac{Mr^2}{2} + M\left(\frac{13r}{2}\right)^2 = \frac{171}{4}mr^2$$

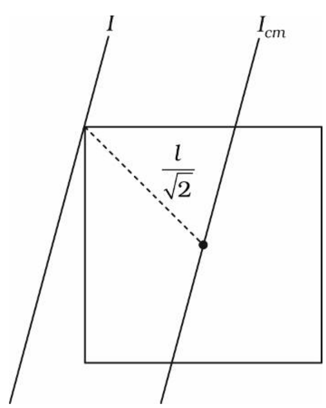
$$I = I_1 + I_2 = 52mr^2$$

18.(B) $I = \frac{Mr^2}{2} + \frac{ML^2}{4}$; $M = \rho\pi r^2 L$; $\qquad I = \frac{\rho\pi r^2 L}{2}\left(r^2 + \frac{L^2}{2}\right)$

$$2.7 = \frac{\rho \times 3.14 \times 0.2^2 \times 0.8}{2}\left(0.2^2 + \frac{0.8^2}{4}\right)$$

$$\rho = \frac{2.7 \times 2}{3.14 \times 0.04 \times 0.8 \times 0.36} = 1.49 \times 10^2\ kg/m^3$$

19.(B)



$$I = I_{cm} + M\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= \frac{Ml^2}{6} + \frac{Ml^2}{2} \qquad = \frac{2}{3}Ml^2$$

20.(C) As per conservation of angular momentum,

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

Initial kinetic energy - final kinetic energy = Loss

$$\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}(I_1 + I_2)\left[\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right]^2 = Loss$$

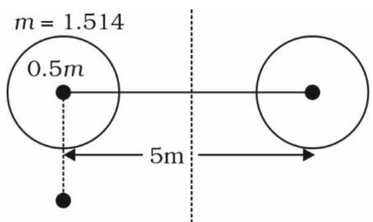
$$\frac{1}{2}\left[I_1\omega_1^2 + I_2\omega_2^2 - \frac{I_1^2\omega_1^2 + I_2^2\omega_2^2 + 2I_1I_2\omega_1\omega_2}{I_1 + I_2}\right]$$

$$= \frac{1}{(I_1 + I_2)} \times \frac{1}{2}\left[I_1^2\omega_1^2 + I_1I_2\omega_2^2 + I_1I_2\omega_1^2 + I_2^2\omega_2^2 - I_1^2\omega_2^2 - 2I_1I_2\omega_1\omega_2\right]$$

$$= \frac{I_1I_2}{2}\left[\frac{\omega_2^2 + \omega_1^2 - 2\omega_1\omega_2}{I_1 + I_2}\right] = \frac{I_1I_2}{(I_1 + I_2)2} \times (\omega_1 - \omega_2)^2$$

21.(Bonus) Point about which angular momentum should be calculated is not mentioned.

22.(D)



$$I = \left(\frac{2}{5}mR^2 + \frac{mL^2}{4}\right) \times 2 = \left(\frac{2}{5} \times 1.5 \times 0.5 \times 0.5 + 1.5 \times \frac{5 \times 5}{4}\right) \times 2$$

$$= (0.15 + 9.375) \times 2 = 9.525 \times 2 = 19.05 kgm^2$$

23.(C) $K_{Rot} = \frac{1}{2}K_{Trans} \quad \Rightarrow \quad \frac{1}{2}mk^2\omega^2 = \frac{1}{4}m\omega^2R^2 \quad \Rightarrow \quad k = \frac{R}{\sqrt{2}}$

$$I_{cm} = mk^2 = \frac{mR^2}{2}$$

24.(4) $\tau = \frac{\Delta L}{\Delta t} = \frac{I(\omega_f - \omega_i)}{\Delta t}$

$$\tau = \frac{\frac{mR^2}{2} \times [0 - \omega]}{\Delta t} = \frac{10 \times (20 \times 10^{-2})^2}{2} \times \frac{600 \times \pi}{30 \times 10} = 0.4\pi = 4\pi \times 10^{-2}$$

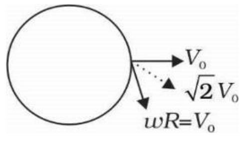
25.(B) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$

$$v = \sqrt{\frac{2gR}{1 + \frac{I}{mR^2}}}$$

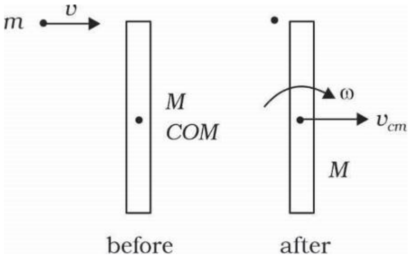
$$I_{ring} > I_{cylinder} > I_{sphere}$$

$$V_{sphere} > V_{cylinder} > V_{ring}$$

26.(2)



27.(4) Let consider before and after situation



Applying conservation of linear momentum

$mv = Mv_{cm}$ ... (i)

Applying conservation of angular momentum about an observer along the velocity of particle at rest w.r.t. ground

$$O = M\ V_{cm}\frac{l}{2}^{\odot} + \frac{Ml^2}{12}\omega^{\otimes}$$

$\Rightarrow \quad v_{cm} = \frac{\omega l}{6}$ ... (ii)

Using coefficient of restitution at the point of collision.

$$1 = \frac{\frac{\omega l}{2} + v_{cm}}{v}$$

$v_{cm} + \frac{\omega l}{2} = v$ ... (iii)

On solving all the 3 equation

$\frac{m}{M} = \frac{1}{4}$ so $x = 4$

28.(3) $v = \sqrt{\dfrac{2gh}{1+\dfrac{K^2}{R^2}}}$; $\dfrac{V_1}{V_2} = \sqrt{\dfrac{1+\frac{1}{2}}{1+1}} = \dfrac{\sqrt{x}}{2} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{x}}{2}$ $\quad\therefore\quad x = 3$

29.(D) Moment of inertia will be different for different axes and so radius of gyration will be different for the three axes. So Assertion A is not correct.

30.(10) $\vec{r} = 10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$; $\vec{v} = \frac{d\vec{r}}{dl} = 20\alpha t\hat{i} + 5\beta\hat{j}$

$\vec{L} = \vec{r}\times m\vec{v} = m\left[50\alpha\beta t^2\hat{k} - 5\beta(t-5)\times 20\alpha t\hat{k}\right]$ or $\vec{L} = 50\alpha\beta mt(t - 2t + 10)\hat{k}$

$L_{t=0} = L_{t=t} \Rightarrow 0 = 50\alpha\beta mt(10-t) \Rightarrow t = 10\,\text{sec}$

31.(A) We have

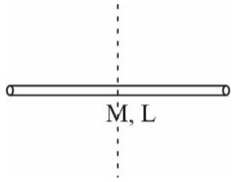
$\frac{M_{Big}}{\pi R^2} = \frac{M_{small}}{\pi r^2}$ (given)

$\frac{M_{Big}}{R^2} = \frac{M_{small}}{r^2} = k$ (let)

$M_{Big} = kR^2$; $M_{small} = kr^2$

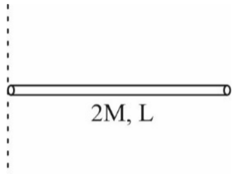
$$\frac{I_{Big}}{I_{small}} = \frac{\frac{M_{big}R^2}{2}}{\frac{M_{small}\times r^2}{4}} = \frac{\frac{kR^4}{2}}{\frac{kr^4}{4}} = 2R^4 : r^4$$

32.(A) (a)



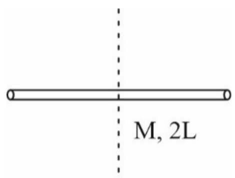
$MOI = \frac{ML^2}{12}$

(b)



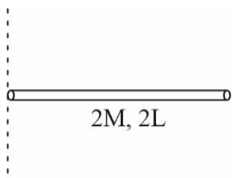
$MOI = \frac{2ML^2}{3}$

(c)



$MOI = \frac{M(2L)^2}{12} = \frac{ML^2}{3}$

(d)



$MOI = \frac{2M(21)^2}{3} = \frac{8ML^2}{3}$

33.(9) $V_P = V_Q$

$w_P \times 3R = w_Q \times R$

$w_Q = 3w_P$

$\frac{1}{2} I_P w_P{}^2 = \frac{1}{2} I_Q w_Q{}^2$; $\quad \frac{I_P}{I_w} = 9$

34.(2) When disc slips, acceleration $a_1 = g \sin\theta$ {θ is angle of inclination}

So time $t_1 = \sqrt{\frac{2L}{a_1}} = \sqrt{\frac{2L}{g \sin\theta}}$

When disc rolls, acceleration : $a_2 \sin\theta \left(\frac{R^2}{R^2 + K^2} \right) = g \sin\theta \left(\frac{2}{3} \right)$

So time taken $t_2 = \sqrt{\frac{2L}{a_2}}$

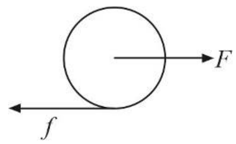
$t_2 = \sqrt{\frac{2L \times 3}{2g \sin\theta}}$

Then $\frac{t_2}{t_1} = \sqrt{\frac{3}{2}}$ hence $x = 2$

JEE Advanced 2021

1.(BD)

(A)



$F - f = ma_{cm}$; $fR = I\alpha$

$a_{cm} = R\alpha$; $F = \frac{Ia_{cm}}{R^2} = ma_{cm}$

$a_{cm} = \frac{F}{I/R^2 + m}$

(B) $fR = \frac{IF}{\left(\frac{I}{R^2} + m\right)R}$

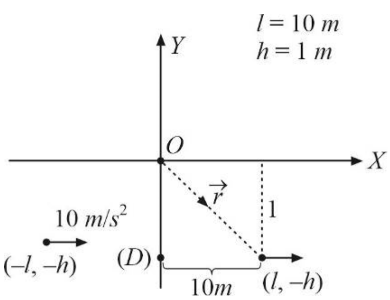
$f = \frac{F}{I + mR^2}$

Depends on F

(C) For thin walled cylinder $I = mR^2, a_{cm} = \frac{F}{2m}$

(D) $\mu\, mg\, R = I\alpha$ $\alpha = \frac{\mu mgR \times 2}{mR^2}$; $\alpha = \frac{2\mu g}{R}$

2.(ABC)



(A) $20 = ut + \frac{1}{2}at^2$

$20 = 0 \times t + \frac{1}{2} \times 10 \times t^2$

$4 = t^2 \Rightarrow t = 2\,\text{sec}$

(B) $\vec{\tau} = \vec{r} \times \vec{F}$

$\Rightarrow \tau = 1 \times 0.2 \times 10\hat{k} = 2\hat{k}$

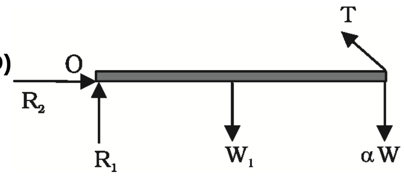
(C) $\vec{\tau} = \vec{r} \times \overline{mv}$

$v = u + at = 10 \times 2 = 20\ m/s\ \hat{i}$

$\vec{L} = 1 \times 0.2 \times 20 = 4\hat{k}$

(D) At D $\vec{\tau} = \vec{h} \times \vec{F}$; $1 \times 0.2 \times 10 = 2\hat{k}$

3.(ABD)



$$\sum \tau_0 = 0 \quad \Rightarrow \quad W\left(\frac{L}{2}\right) + \alpha W(L) - \frac{T}{\sqrt{2}}(L) = 0$$

$$\Rightarrow \quad \frac{T}{\sqrt{2}} = W\left(\alpha + \frac{1}{2}\right) \quad \Rightarrow \quad \boxed{T = \sqrt{2}W\left(\alpha + \frac{1}{2}\right)}$$

$$\sum F_y = 0$$

$$\Rightarrow \quad R_1 + \frac{T}{\sqrt{2}} = W + \alpha W$$

$$\Rightarrow \quad R_1 + \alpha W + \frac{W}{2} = W + \alpha W$$

$$\Rightarrow \quad \boxed{R_1 = \frac{W}{2}}$$ Option A is correct

$$R_2 = \frac{T}{\sqrt{2}} = W\left(\alpha + \frac{1}{2}\right)$$

When $\alpha = 0.5$, $R_2 = W$ Option B is correct

$\alpha = 0.5$, $T = \sqrt{2}W(0.5 + 0.5) = \sqrt{2}W$ Option C is incorrect

$\alpha > 1.5 \Rightarrow$ $T > 2\sqrt{2}W$, Hence rope will break

Option D is correct

4.(49) $$\frac{Ma^2}{3}\omega + \frac{M\left(\frac{a}{4}\right)^2}{2}4\omega + M\left(\frac{3a}{4}\right)^2\omega$$

$$= \frac{Ma^2\omega}{3} + \frac{Ma^2\omega}{8} + \frac{9Ma^2\omega}{16}$$

$$= \frac{16Ma^2\omega + 6Ma^2\omega + 27Ma^2\omega}{48} = \frac{49Ma^2\omega}{48}$$

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Solutions of Archive - JEE Main & Advanced

Rotational Motion	Class - XI | Physics

JEE Main 2022

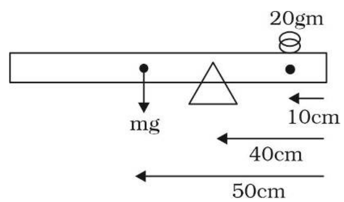
1.(6) Balancing torques about knife edge

$mg(10) = (20 \times 10^{-3})g \times 30$

$m = 60 \times 10^{-3}$

$= 0.06 kg$

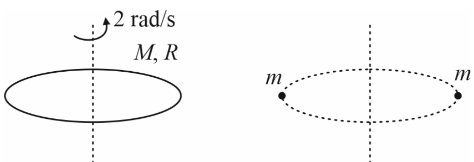
$m = 6 \times 10^{-2}$; $x = 6$



2.(5) $2\left(\frac{1}{2}MR^2 + \frac{1}{4}MR^2\right) + \frac{1}{2}MR^2 = x\frac{2}{5}MR^2$

$2MR^2 = x\frac{2MR^2}{5} \Rightarrow x = 5$

3.(C)



$MR^2(2) = (MR^2 + 2mR^2)\omega'$; $\omega' = \frac{M \times 2}{M + 2m} = \frac{2m}{M + 2m}$

4.(B) $K_{rotation} = \frac{1}{2}I_{CM}\omega^2 = \frac{1}{2} \times \frac{2}{5}MR^2 \times \omega^2 = \frac{1}{5}mv^2$

$K_{Total} = K_{rotation} + \frac{1}{2}mv^2 = \frac{1}{5}mv^2 + \frac{1}{2}mv^2 = \frac{7}{10}mv^2$ $\therefore$ $\frac{K_{rotation}}{K_{total}} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2} = \frac{2}{7}$

5.(3) By conservation of energy,

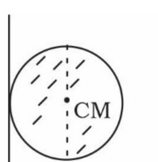
$K_i + U_i = K_f + U_f$

$12gh = 3gh + \frac{1}{2}12(1+1)V_{cm}^2 + \frac{1}{2}3V_{cm}^2$

$9gh = \frac{1}{2}V_{cm}^2[24+3]$; $9gh = \frac{27}{2}V_{cm}^2$

$V_{cm} = \sqrt{\frac{2}{3}gh} = \frac{1}{2}\sqrt{\frac{8}{3}gh} = \frac{1}{2}\sqrt{2.67gh} \Rightarrow x = 2.67$

6.(A) (A)

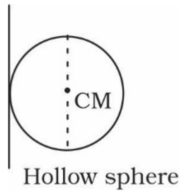


Solid sphere

For solid sphere

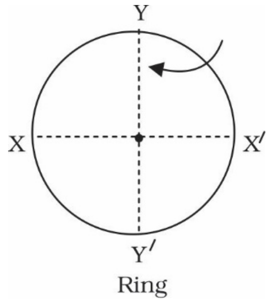
$I = I_{cm} + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

(B)



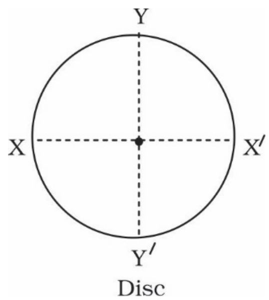
$I = I_{cm} + MR^2 = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$

(C)



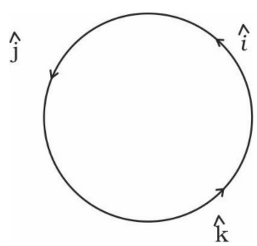
$I_{xx'} + I_{yy} = I_{zz}$ Or $I + I = MR^2 \Rightarrow I = \frac{MR^2}{2}$

(D)



$I_{xx'} + I_{yy'} = I_{zz'} \Rightarrow I + I = \frac{MR^2}{2} \quad \Rightarrow I = \frac{MR^2}{4}$

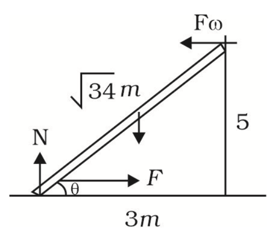
7.(91)



$\vec{L} = \vec{r} \times m\vec{v} = \left(3\hat{i} - \hat{j}\right) \times \left(3\hat{j} + \hat{k}\right) = 9\hat{k} + 3\left(-\hat{j}\right) - 0 - \hat{i} = -\hat{i} - 3\hat{j} + 9\hat{k}$

$\left|\vec{L}\right| = \sqrt{1^2 + 3^2 + 9^2} = \sqrt{91}$

8.(C) $f = F_\omega$; $N = mg = 100kg$



$F_\omega \times \sqrt{34}\sin\theta = \frac{mg}{2}\sqrt{34}\cos\theta$

$F_\omega = \frac{mg}{2}\frac{\cos\theta}{\sin\theta} = \frac{mg}{2}\cot\theta$

$F_\omega = \frac{100}{2} \times \frac{3}{5}$; $F_\omega = 30N$

$F_f = \sqrt{N^2 + F^2}$

$F_f = \sqrt{\left((30)^2 + (100)^2\right)} = \sqrt{900 + 1000}$

$= \sqrt{10900} = 10\sqrt{109}N$; $\frac{F\omega}{F_f} = \frac{30}{10\sqrt{109}} = \frac{3}{\sqrt{109}}$

9.(C) $L_0 = m(\vec{r}_{CM} \times \vec{v}_{CM}) + I_{CM}\vec{w}$

$= mvR + \frac{2}{3}mR^2w$

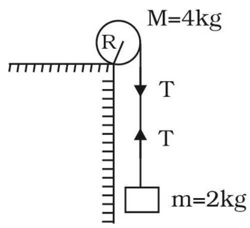
$= mR^2w + \frac{2}{3}mR^2w = \frac{5}{3}mR^2w$; $a = 5$

10.(8) $I_1 = \frac{m\ell^2}{3}$ and $\ell = 2\pi r$

$I_2 = \frac{mr^2}{2}$

$\frac{I_1}{I_2} = \frac{2}{3}\left(\frac{\ell}{r}\right)^2 = \frac{2}{3} \times 4\pi^2 = 8\frac{\pi^2}{3}$ $\therefore$ $x = 8$

11.(10) $\tau = I\alpha$

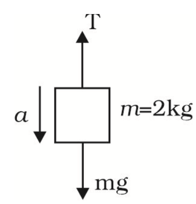


$TR = \frac{MR^2}{2}\alpha$ ...(i)

For pure rolling $a = \alpha R$

$\Rightarrow$ $TR = \frac{MR^2}{2}\left(\frac{a}{R}\right)$

$\Rightarrow$ $\boxed{T = 2a}$ ...(ii)



$mg - T = ma$

$\Rightarrow$ $20 - 2a = T$

$\Rightarrow$ $20 - \frac{2T}{2} = T$

$\Rightarrow$ $20 = 2T$ $\Rightarrow$ $(T = 10N)$

12.(D) In general, $V = \sqrt{\frac{2gh}{1 + k^2 / R^2}}$

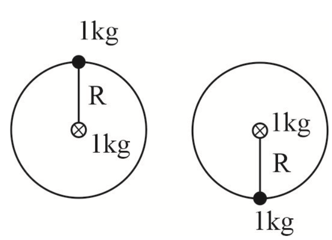
Where k is radius of gyration

$$\frac{V_{\text{solid cylinder}}}{V_{\text{solid sphere}}} = \sqrt{\frac{\left(1 + \frac{k^2}{R^2}\right)_{\text{solid sphere}}}{\left(1 + \frac{k^2}{R^2}\right)_{\text{solid cylinder}}}}$$

$$= \sqrt{\frac{1 + \frac{2}{5}}{1 + \frac{1}{2}}} \qquad = \sqrt{\frac{7}{5} \times \frac{2}{3}} = \sqrt{\frac{14}{15}}$$

13.(5) Loss in P.E. = gain in K.E.

$\Rightarrow$ $(1kg)(10)(2R) = \frac{1}{2}I_H\omega^2$

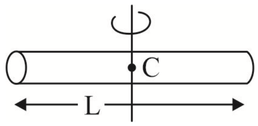


$\Rightarrow$ $20R = \frac{1}{2}.\left(\frac{1}{2}mR^2 + mR^2\right)\omega^2$

$\Rightarrow$ $80 = 3mR\omega^2$

$\omega = \sqrt{\frac{80}{3R}} = 4\sqrt{\frac{5}{3R}} = 4\sqrt{\frac{x}{3R}}$ $\therefore$ $x = 5$

14.(5) $I = \dfrac{ML^2}{12}$



Radius of gyration

$k = \sqrt{\dfrac{I}{M}} = \sqrt{\dfrac{L^2}{12}}$

$k = \sqrt{\dfrac{(10\sqrt{3})^2}{12}} = 5$

15.(18) $\tau = rF$; $\tau = \dfrac{3}{2} \times (12t - 3t^2)$

$\tau = I\alpha$; $\dfrac{3}{2}(12t - 3t^2) = \dfrac{3}{2}\alpha$

$4t - t^2 = \alpha$; $\quad 4t - t^2 = \dfrac{d\omega}{dt}$

$\omega = 2t^2 - \dfrac{t^3}{3}$

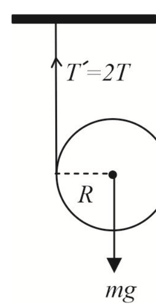
$\omega = 0$ when it is changing direction

$t = 6\,\text{sec}$; $\quad \theta = \dfrac{2}{3}t^3 - \dfrac{t^4}{12}$

$\theta = t^3\left(\dfrac{2}{3} - \dfrac{t}{12}\right) = 6^3\left(\dfrac{2}{3} - \dfrac{1}{2}\right)$

$\theta = 6^3 \times \dfrac{1}{6}$; $\quad \theta = 36$; $\eta = \dfrac{\theta}{2\pi} = \dfrac{18}{\pi}$

16.(120) $mg - T' = ma_{cm}$ ... (i)



$\tau_{cm} = T'R = I_{cm}\alpha = \dfrac{MR^2}{2} \times \alpha$

$\Rightarrow \quad T' = \dfrac{MR\alpha}{2} = \dfrac{M}{2} \times a_{cm}$ ... (ii)

Equation (i) and (ii)

$mg = \left(m + \dfrac{m}{2}\right)a_{cm}$; $\quad a_{cm} = \dfrac{2}{3}g$

$V_{cm}^2 = u_{cm}^2 + 2a_{cm}s_{cm}$; $\quad (4)^2 = 0 + 2 \times \dfrac{2 \times 10}{3} \times s_{cm}$

$s_{cm} = \dfrac{16 \times 3}{4 \times 10} = \dfrac{12}{10} = 1.2m = 120\,cm$

17.(3) $I = 2\left(\dfrac{M\left(\dfrac{a}{2}\right)^2}{4}\right) + 2\left(\dfrac{M(a/2)^2}{4} + M\left(\dfrac{a}{2}\right)^2\right)$

$= \dfrac{Ma^2}{8} + \dfrac{Ma^2}{8} + \dfrac{Ma^2}{2} \quad = \dfrac{3}{4}Ma^2$

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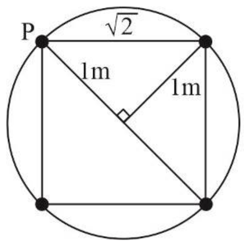
1.(B)



$F_{net} = F + \sqrt{2}F'$; $F = \dfrac{Gm^2}{(2)^2}$ & $F' = \dfrac{Gm^2}{\left(\sqrt{2}\right)^2}$

$$\Rightarrow \quad F_{net} = \frac{Gm^2}{4} + \sqrt{2}\frac{Gm^2}{2} = \left(\frac{2\sqrt{2}+1}{4}\right)Gm^2 = \frac{mV^2}{(1)}$$

$$= V = \frac{\sqrt{G\left(2\sqrt{2}+1\right)}}{2} \qquad (\because\ m = 1kg)$$

2.(C) $F = \mu\omega^2 d = \dfrac{G(m)(2m)}{d^2}$

Where $\mu = \dfrac{m(2m)}{3m}$ (reduced mass of m and 2m)

$$\Rightarrow \quad \omega^2 = \frac{3Gm}{d^3} \quad \Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

3.(D) $\omega_1 = 2\pi / T_1$

$\omega_2 = 2\pi / T_2$

$\dfrac{\omega_1}{\omega_2} = \dfrac{T_2}{T_1} = \dfrac{8}{1}$

4.(C) We know that $g_{equator} = g_{pole} - w^2 R$

Here, $w = \dfrac{2\pi}{T} = \dfrac{2\pi}{24\times 60\times 60}$ rad/s

Since weight, $W = \text{mg}$

$W_{\text{equator}} = W_{\text{pole}} - mw^2 R$

Here, mass $m = \dfrac{49}{9.8} = 5$ kg

Note that the calculation is quite cumbersome but we can notice that W_{equator} must be smaller than W_{pole} and only one option matches this condition, and hence we need not calculate.

5.(D) $V_A = \sqrt{\dfrac{GM}{r_A}} = \sqrt{\dfrac{20\times10^{-11}\times6\times10^{24}}{3\times7\times10^{6}}} m/s$

$V_A = \dfrac{2}{\sqrt{7}}\times10^4 m/s$

$T_A = \dfrac{2\pi r_A}{V_A} = \dfrac{2\pi\times7\times10^6\sqrt{7}}{2\times10^4}s = 7\sqrt{7}\pi\times10^2 s = 5.8\times10^3 s$

$T_B = T_A\left(\dfrac{r_B}{r_A}\right)^{3/2} = T_A\dfrac{8\sqrt{8}}{7\sqrt{7}} = 7.1\times10^3 s \quad \therefore \quad T_B - T_A = 1.3\times10^3 s$

6.(A) $V_A = V_B \quad \Rightarrow \quad \sqrt{\dfrac{GM_1}{R_1}} = \sqrt{\dfrac{GM_2}{R_2}} \qquad \therefore \quad M_1R_2 = M_2R_1$

7.(B) $F_1 = \dfrac{GMm}{9R^2}$

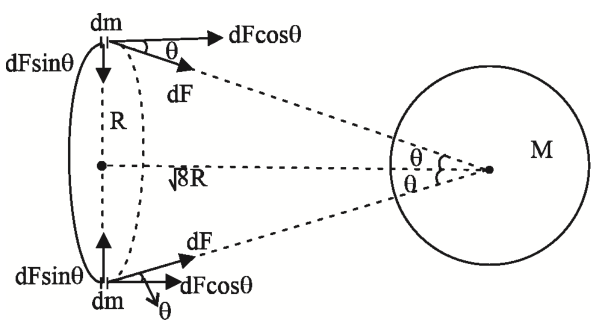
$F_2 = \dfrac{GMm}{9R^2} - \dfrac{GMm/8}{(2.5R)^2} = \dfrac{GMm}{9R^2} - \dfrac{GMm}{50R^2} = \dfrac{41GMm}{450R^2} \qquad \therefore \quad \dfrac{F_1}{F_2} = \dfrac{GMm}{9R^2}\times\dfrac{450R^2}{41GMm} = 50:41$

8.(10) Using energy conservation $\dfrac{1}{2}mv^2 - \dfrac{GMm}{R} = -\dfrac{GMm}{11R}$

$v = \sqrt{\dfrac{20GMe}{11R}} = \sqrt{\dfrac{10}{11}}v_e$; So $x = 10$

9.(B) For planet revolving in elliptical orbit, its areal velocity is constant, by Keplers 2[nd] law so option (B) is correct.

10.(B)



On each element of ring diametrically opposite force components will get cancelled out.

$F_{net} = \int dF\cos\theta = \int\dfrac{GM(dm)}{\left(\sqrt{8R^2+R^2}\right)^2}\times\dfrac{\sqrt{8}R}{\sqrt{(8R^2+R^2)}} = \dfrac{GMm}{27R^2}\cdot\dfrac{\sqrt{8}R}{R} = \dfrac{\sqrt{8}}{27}\dfrac{GMm}{R^2}$

Option (B) is correct.

11.(4) $g_A = \dfrac{GM}{R^3}(OA) = g_B = \dfrac{GM}{(OC)^2} = \dfrac{GM}{(R+R/2)^2} = \dfrac{4GM}{9R^2}$

$AB = R_e - OA = R - \dfrac{4R}{9} = \dfrac{5R}{9}$; $OA : AB = 4:5$ Thus, $x = 4$

12.(64) $V_{esc} = \sqrt{\dfrac{2GM}{R}}$ ... (1)

$10V_{esc} = \sqrt{\dfrac{2GM}{R_{new}}}$ ... (2)

By (1) and (2), $R_{new} = \dfrac{R}{100} = 64km$

13.(D) By conservation of angular momentum

$\cancel{m}v_1r_1 = \cancel{m}v_2r_2$

$\Rightarrow \quad 6\times10^4\times8\times10^{10} = V_2\times1.6\times10^{12} \quad \Rightarrow \quad V_2 = \dfrac{6\times10^4\times\cancel{8}\times10^{10}}{\cancel{1.6}\times10^{12}}$

$= 3\times10^3\ \text{m/s}$

14.(C) $T^2 \propto R^3$; $\quad \left(\dfrac{T}{24}\right)^2 = \left(\dfrac{3R}{12R}\right)^3 \quad \Rightarrow \quad T = 3hr$

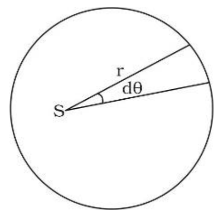
15.(D) $T^2 \propto R^3$

$\therefore \quad \left(\dfrac{T_2}{T_1}\right)^2 = \left(\dfrac{R_2}{R_1}\right)^3 \quad \therefore \quad \left(\dfrac{T_2}{T_1}\right)^2 = 9^3 \quad \therefore \quad \dfrac{T_2}{T_1} = 27$

16.(C) $dA = \dfrac{1}{2}r^2d\theta$

$\dfrac{dA}{dt1} = \dfrac{1}{2}r^2\dfrac{d\theta}{dt}$

$\dfrac{dA}{dt} = \dfrac{1}{2m}mr^2\dfrac{d\theta}{dt} = \dfrac{L}{2m}$

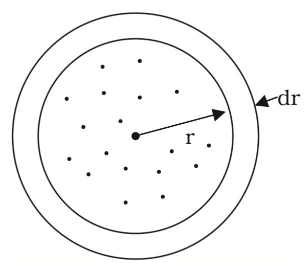


17.(A) $g_{eff} = g - \omega^2\,\text{Re}$ (at equator)

$\omega = \sqrt{\dfrac{g}{\text{Re}}}$; $\quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{\text{Re}}{g}} \approx 84\ \text{min}$

18.(3) Potential energy stored in bringing shell of thickness *dr* over the solid sphere of radius *r* from infinity

$= dU = -\left(\dfrac{GM_r}{r}\right).dm$



$M_r =$ mass of solid sphere of radius r. $= \left(\dfrac{Mr^3}{R^3}\right)$

dm = mass of shell of thickness dr. $= (density)\times4\pi r^2dr = \dfrac{3M}{R^3}.r^2dr$

$M =$ Mass of sphere of radius R.

$\Rightarrow \quad P.E. = -\int\limits_{r=0}^{R}\dfrac{G.\left(Mr^3/R^3\right)}{r}.\dfrac{3M}{R^3}.r^2dr = -\dfrac{3GM^2}{R^6}\int\limits_{r=0}^{R}r^4dr = -\dfrac{3GM^2}{R^6}\times\dfrac{R^5}{5} = -\dfrac{3}{5}\dfrac{GM^2}{R}$

Work to be done $= \dfrac{3GM^2}{5R}$

19.(A) Gravitational field is zero and potential is constant inside a shell.

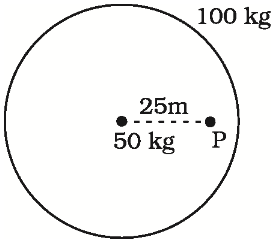
20.(2) $PE = \left\{\dfrac{-Gm(M-m)}{d}\times4\right\} - \dfrac{Gm^2}{\sqrt{2}d} - \dfrac{G(M-m)^2}{\sqrt{2}d}$

For PE to be maximum, $\dfrac{d(PE)}{dm} = 0 \quad \Rightarrow \quad \dfrac{-4G}{d}(M-2m) - \dfrac{G}{\sqrt{2}d}2m - \dfrac{G}{\sqrt{2}d}2(M-m)(-1) = 0$

$\Rightarrow \quad 4(M-2m) + \sqrt{2}m + \sqrt{2}(M-m)(-1) = 0 \quad \Rightarrow \quad 4M - 8m + \sqrt{2}m - \sqrt{2}M + \sqrt{2}m = 0$

$\Rightarrow \quad \left(4-\sqrt{2}\right)M = \left(8-2\sqrt{2}\right)m \quad \Rightarrow \quad \dfrac{M}{m} = 2$

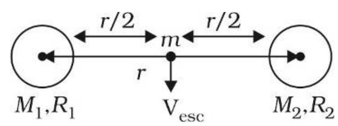
21.(B) r = 50 m



Potential at P $= -\dfrac{G\times 100}{50} - \dfrac{G\times 50}{25} = -2G - 2G = -4G$

22.(B) By mechanical energy conservation

$M.E_i = M.E_f$

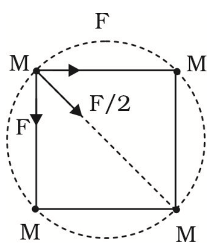


$$-\frac{GM_1m}{(r/2)} - \frac{GM_2m}{(r/2)} + \frac{1}{2}mV_{esc}^2 = 0$$

$$\frac{V_{esc}^2}{2} = \frac{2G}{r}(M_1 + M_2)$$

$$V_{esc} = \sqrt{\frac{4G(M_1+M_2)}{r}}$$

23.(B) Net force on Q no of the mass $= \dfrac{Gm^2}{R^2}\left(\dfrac{1}{4} + \dfrac{1}{\sqrt{2}}\right)$



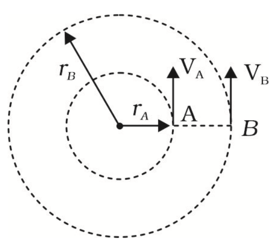
$F = \dfrac{GM^2}{2R^2}$ Now, $= \dfrac{Gm^2}{R^2}\left(\dfrac{1}{4} + \dfrac{1}{\sqrt{2}}\right) = \dfrac{mv^2}{R}$

$$V = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1 + 2\sqrt{2}\right)}$$

24.(3) as $T^2 \alpha R^3$

$T_A^2 \alpha r_A^3$

$T_B^2 \alpha r_B^3$



Now, $\dfrac{T_A}{T_B} = \left(\dfrac{r_A}{r_B}\right)^{3/2}$

$\Rightarrow r_B = 4r_A$

$r_B = 4 \times 2 \times 10^3 km$

$$W_{AB} = \frac{(V_B - V_A)}{r_B - r_A} = \frac{\left|2\pi\left(\frac{r_B}{T_B} - \frac{r_A}{T_a}\right)\right|}{r_B - r_A} = \frac{\pi}{3}$$

$\Rightarrow \dfrac{\pi}{x} = \dfrac{\pi}{3} \quad x = 3$

25.(A) For at height r

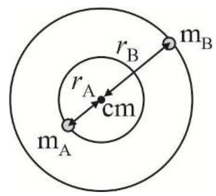
$$g' = \frac{Gm}{(R_E + r)^2} = \frac{GM}{R_E^2 + r^2 + 2R_E r}$$

For at depth r

$$g'' = \frac{GM(R_E - r)}{R_E^3}; \qquad g'' = \frac{GM}{R_E^2}\left[1 - \frac{r}{R_E}\right]$$

$$\frac{g''}{g'} = \frac{\frac{GM}{R_E^2}\left[1 - \frac{r}{R_E}\right]}{\frac{GM}{R_E^2 + r^2 + 2R_E r}}; \qquad \frac{g''}{g'} = 1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$$

26.(B) For c.m. of binary star to be at rest, angular speeds must be equal.



$\because \quad \omega_A = \omega_B$

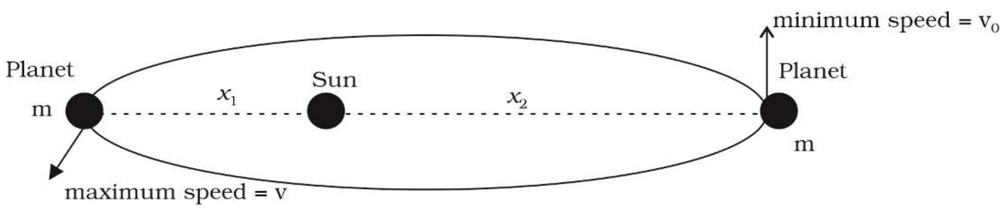
$\Rightarrow \quad \dfrac{2\pi}{T_A} = \dfrac{2\pi}{T_B}$

$\therefore \quad T_A = T_B$

27.(D) $T^2 \propto R^3 \Rightarrow 2\dfrac{\Delta T}{T} = 3\dfrac{\Delta R}{R} \Rightarrow \dfrac{\Delta T}{T} = \dfrac{3}{2}\left(\dfrac{0.02R}{R}\right) \Rightarrow \dfrac{\Delta T}{T} = 0.03$

% difference = $0.03 \times 100\% = 3\%$

28.(C)



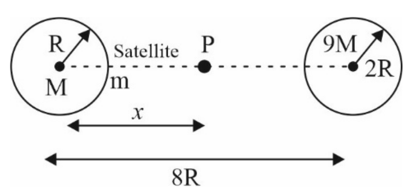
Conservation of angular momentum $\Rightarrow mvx_1 = mv_0 x_2 \Rightarrow v = v_0\left(\dfrac{x_2}{x_1}\right)$

29.(D) Mass = Volume × Density

$$\frac{M_p}{M_e} = \frac{V_p\, d_p}{V_e\, d_e} \quad \Rightarrow \quad \frac{V_p}{V_e} = 2$$

$\therefore \quad \dfrac{R_p}{R_e} = 3\sqrt{2} \qquad$ Now $\quad g = \dfrac{GM}{R^2} = \dfrac{4}{3}\pi G d R$ and $\dfrac{W_p}{W_e} = \dfrac{g_p}{g_e} = \dfrac{R_p}{R_e} = 2^{1/3}$

30.(4) At P force is O



$$\frac{GMm}{x^2} = \frac{G(9M)m}{(8R - x)^2} \Rightarrow (8R - x)^2 = 9x^2$$

$\Rightarrow 8R - x = 3x \Rightarrow 8R = 4x \Rightarrow x = 2R$

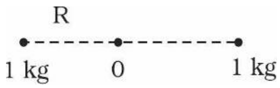
Mechanical energy conservation between surface of M to P

$$\frac{-GMm}{R} - \frac{G(9M)m}{7R} + \frac{1}{2}mv_{min}^2 = \frac{-GMm}{2R} - \frac{G(9M)m}{6R}$$

$$\frac{1}{2}mv_{min}^2 = \frac{GMm}{R}\left(\frac{1}{1} + \frac{9}{7} - \frac{1}{2} - \frac{9}{6}\right)$$

$$\frac{V_{min}^2}{2} = \frac{GM}{R}\left(\frac{42 + 54 - 21 - 63}{42}\right) \;;\quad v_{min}^2 = \frac{GM}{R} \times \frac{12}{21} \quad \Rightarrow \quad v_{min} = \sqrt{\frac{4}{7}\frac{GM}{R}} \Rightarrow a = 4$$

31.(C)



$$F = \frac{G(1)(1)}{(2R)^2} = m\omega^2(R) \quad ; \quad \omega = \left(\frac{G}{4R^3}\right)^{1/2}$$

32.(B) $T = \frac{2\pi r\sqrt{r}}{\sqrt{GM}}$; $M = \frac{4\pi^2 r^3}{T^2 G}$; $r = 9\times10^6 m$

$T = 7.5\times3600\,\text{sec} \qquad \therefore \qquad M = 6\times10^{23} kg$

33.(C) As $g \alpha \frac{1}{R^2}$ so graph will be curve

Also, there will be an instant when

$F_{\text{earth}} = F_{\text{mars}}$

So graph (c)

34.(A) By conservation of energy

$$0 = \frac{1}{2}mv^2 - \frac{GMm}{(R+x)}$$

$$\frac{v^2}{2} = \frac{gR^2}{R+x}; \qquad V^2 = \frac{2gR^2}{R+x}; \qquad V = \sqrt{\frac{2gR^2}{R+x}}$$

$$\frac{dx}{dt} = \sqrt{\frac{2gR^2}{R+x}} \Rightarrow \int_0^h \sqrt{R+x}\,dx = \sqrt{2gR^2}\int_0^t dt \Rightarrow \qquad \left[\frac{(R+x)^{3/2}}{3/2}\right]_0^h = \sqrt{2gR}\ t$$

$$\Rightarrow \qquad \frac{2}{3}(R+h)^{3/2} - R^{3/2} = \sqrt{2g}\ Rt \qquad \Rightarrow \qquad t = \frac{1}{3}\sqrt{\frac{2R}{g}}\left[\left(1+\frac{h}{R}\right)^{3/2} - 1\right]$$

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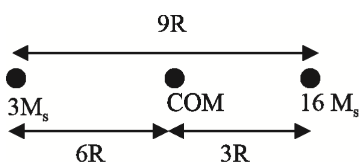
1.(9)



Let velocity of $3M_S$ star be V_1

$$\frac{(3M_s)V_1^2}{6R} \qquad \frac{G(3M_s)(6M_s)}{(9R)^2}$$

$$\frac{3M_S V_1^2}{6R} = \frac{18GM_S^2}{81R^2} \Rightarrow V_1^2 = \frac{4}{9}\frac{GM_S}{R} \Rightarrow V_1 = \frac{2}{3}\sqrt{\frac{GM_S}{R}}$$

Time period of revolution

$$nT = \frac{2\pi(6R)}{V_1} = \frac{2\pi\times6R}{\frac{2}{3}\sqrt{\frac{GM_S}{R}}} \Rightarrow nT = 18\pi R\sqrt{\frac{R}{GM_S}}$$

$$nT = \frac{18\pi\times R^{3/2}}{\sqrt{GM_S}}$$

Time period of earth's revolution, $T = \frac{2\pi\times R^{3/2}}{\sqrt{GM_S}}$

$n = 9$

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1.(B) $Mg' = \frac{M}{3}g$; $\quad g' = \frac{g}{3}$

$$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{3}$$

$\frac{R}{R+h} = \frac{1}{\sqrt{3}}$; $\quad h = \left(\sqrt{3}-1\right)R \quad = (1.732-1)6400$; $\quad h = 4685\,km$

2.(D) According to Kepler's law time period,

$T^2 \propto R^3$, we have

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} \quad \Rightarrow T_2 = 3\sqrt{3}T_1$$

$\therefore$ The duration of year if distance between sun and earth becomes 3R will be $3\sqrt{3}$ years.

3.(D) $r = 3R$; $\quad g' = \frac{GM}{(3R)^2} = \frac{g}{9}$

4.(2) The speed of revolution is given by

$$V = \sqrt{\frac{GM}{r}} \quad \Rightarrow V \propto \frac{1}{\sqrt{r}}$$

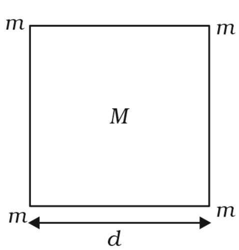
$$\frac{V_1}{V_2} = \sqrt{\frac{r_2}{r_1}} \quad \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$$

5.(A)

6.(D) Magnitude of g varies as we move from pole to equator because of variation in the radius of earth.

7.(A) Both statement are true as per definition.

8.(A)

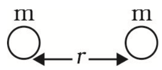


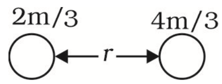
$$U = -\frac{4Gm^2}{d} - 2\frac{Gm^2}{\sqrt{2}d} - 4\frac{GmM}{d/\sqrt{2}} \quad \Rightarrow \quad U = -\frac{Gm}{d}\left\{\left(4+\sqrt{2}\right)m + 4\sqrt{2}M\right\}$$

9.(C) $(T_A)^2 = (2T_B)^2$

$(r_A)^3 = 4 \times (r_B)^3 \quad \left[\because T^2 \propto r^3\right]$

10.(C) $F = \dfrac{Gm^2}{r^2}$





$$F' = \dfrac{\dfrac{8Gm^2}{9}}{r^2} = \dfrac{8}{9}\dfrac{Gm^2}{r^2}$$

$$F' = \dfrac{8}{9}F$$

11.(A) $V_e = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2G \times \rho \times \frac{4}{3}\pi R^3}{R}} \propto \sqrt{\rho R^2}$

$$\dfrac{v_A}{v_B} = \dfrac{12}{v_B} = \dfrac{\sqrt{\rho R^2}}{\sqrt{4\rho\left(\dfrac{R}{2}\right)^2}} \Rightarrow \quad v_B = 12 km\ /\ s$$

12.(B) $T_1 = 7$ hrs; $\quad r_2 = 3r_1$

$\because \quad T^2 \propto r^3$; $\quad \dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$; $\quad \dfrac{7^2}{T_2^2} = \left(\dfrac{r_1}{3r_1}\right)^3$; $\quad \dfrac{49}{T_2^2} = \dfrac{1}{27}$

$T_2 = \sqrt{27 \times 49}$; $\quad T_2 = 3 \times 7\sqrt{3}$

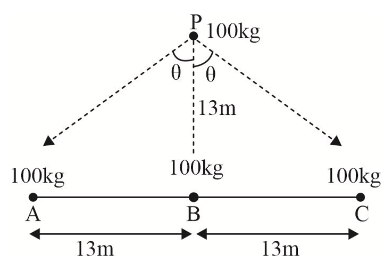
$T_2 = 21 \times \sqrt{3}$; $\quad T_2 \cong 36$ hours

13.(B) $F_{net} = F + 2F\cos\theta$

$$= \dfrac{GMm}{13^2} + \dfrac{2GMm}{\left(13\sqrt{2}\right)^2}\left(\dfrac{1}{\sqrt{2}}\right)$$

$$= \dfrac{G10^4}{13^2}\left(1 + \dfrac{1}{\sqrt{2}}\right)$$

$$\approx 100G$$



14.(A) % decrease in g = % decrease in weight

$$= \dfrac{g_{\text{surface}} - g_{\text{height}}}{g_{\text{surface}}} \times 100$$

$$= \dfrac{\dfrac{GM}{R^2} - \dfrac{GM}{(R+h)^2}}{\dfrac{GM}{R^2}} \times 100; \quad \left(h = \dfrac{R}{4}\right)$$

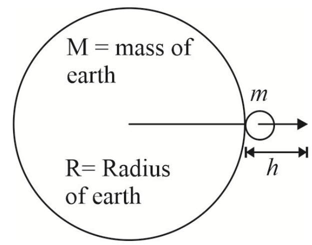
$$= \dfrac{\dfrac{1}{R^2} - \dfrac{1}{\left(\dfrac{5R}{4}\right)^2}}{1/R^2} \times 100 \qquad = \left(1 - \dfrac{16}{25}\right) \times 100 \ = \dfrac{9}{25} \times 100 = 36\%$$

15.(A) % decrease in $g = \left(\dfrac{2h}{R}\right) \times 100\%$

$$= \left(\dfrac{32 \times 2}{6400}\right) \times 100\%$$

$$= \left(\dfrac{1}{100}\right) \times 100\% = 1\%$$

16.(A) Escape speed $= \sqrt{\frac{2GM}{R}}$



$V =$ speed of body $= \frac{1}{3}\sqrt{\frac{2GM}{R}}$

Initial Kinetic energy of body at projection

$$= \frac{1}{2}mV^2 = \frac{1}{2}m\frac{1}{9}\left(\frac{2GM}{R}\right) = \frac{GMm}{9R}$$

Initial potential energy of earth surface $= \frac{-GMm}{R}$

Final kinetic energy at top most point = 0

Final potential energy $= -\left(\frac{G\ Mm}{R+h}\right)$

According to law of conservation of energy

Initial total energy = Final total energy

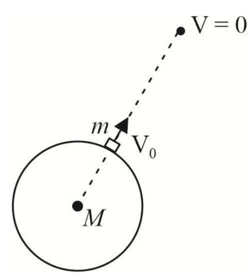
$$-\frac{GMm}{R} + \frac{GMm}{9R} = \frac{-GMm}{R+h}; \qquad -\frac{1}{R} + \frac{1}{9R} = \frac{-1}{R+h}$$

$$\frac{-9+1}{9R} = \frac{-1}{R+h} = \frac{8}{9R} = \frac{1}{R+h}$$

$$8R + 8h = 9R; \quad 8h = 9R - 8R; \quad h = \frac{6400\ \text{km}}{8} \ \boxed{h = \frac{R}{8}}$$

17.(B) $E = -\frac{1}{2}\frac{GMm}{r}$; $\quad \frac{E_1}{E_2} = \left(\frac{m_1}{m_2}\right) \times \left(\frac{r_2}{r_1}\right) = \frac{4}{3} \times \frac{4r}{3r} = 16:9$

18.(B) $\frac{-GMm}{R} + \frac{1}{2}mv_0^2 = -\frac{GMm}{h} + 0$



$$\frac{-GMm}{R} + \frac{1}{2}m \times \lambda^2 \times \frac{2GM}{R} = \frac{-GMm}{h} \qquad \left[V_e = \sqrt{\frac{2GM}{R}}\right]$$

$$\Rightarrow \qquad \frac{1}{h} = \frac{1}{R} - \frac{\lambda^2}{R}; \qquad h = \frac{R}{(1-\lambda^2)}$$

19.(D) $g = \frac{GM}{R^2}$; $\qquad \frac{\Delta g}{g} = -2\left(\frac{\Delta R}{R}\right)$

20.(2) For $h << \text{Re}$

$$g\left(1 - \frac{2h}{R}\right) = g\left[1 - \frac{\alpha h}{R}\right] \quad \therefore \qquad \alpha = 2$$

21.(A) $\Delta U = U_f - U_i$

$$= \left(-\frac{GMm}{R+3R}\right) - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMm}{R}\left(1 - \frac{1}{4}\right)$$

$$= \frac{3}{4}\frac{GMm}{R} \times \frac{R}{R} = \frac{3}{4}gmR = \frac{3}{4} \times 10 \times 1 \times 6400 \times 10^3$$

$$= 48 \times 10^6\ J \qquad = 48\,MJ$$

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Liquids	Class - XI | Physics

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1.(25600)

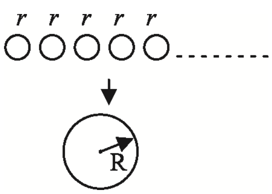
Using pascal's law ; $\dfrac{100g}{A_1} = \dfrac{mg}{A_2}$

After the changing of cross-sectional areas,

Let 'x' be the mass lifted

$\therefore \quad \dfrac{xg}{16A_1} = \dfrac{mg}{\left(\dfrac{A_2}{16}\right)} \quad \therefore \quad xg = 100 \times 16 \times 16g \quad \therefore \quad x = 25600kg$

2.(B)



$NV_S = V_L = V$ $\quad \{V_s \to$ volume of small drops

$\{V_L \to$ volume of large drops

Initial surface energy $U_i = NTA_i = T4\pi r^2 N$

Final surface energy $U_F = TA_F = T4\pi R^2$

Decrease in surface energy $= NT4\pi r^2 - T4\pi R^2$

$$= T\left[\frac{N4\pi r^2 3r}{3r} - \frac{4\pi R^2 3R}{3R}\right] = 3T\left[\frac{\frac{N4\pi r^3}{3}}{r} - \frac{4\pi R^3}{3R}\right] = 3T\left[\frac{V}{r} - \frac{V}{R}\right] \quad \left(V = N\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3\right)$$

Decrease in surface energy $= 3TV\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$

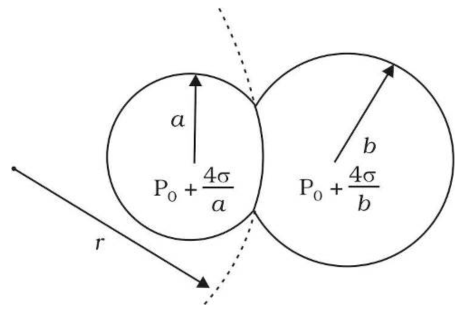
$Q = \dfrac{W}{J} \quad \Rightarrow \quad Q = \dfrac{3TV}{J}\left[\dfrac{1}{r} - \dfrac{1}{R}\right] \quad \Rightarrow \quad \dfrac{Q}{V} = \dfrac{3T}{J}\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$

3.(C) For common surface meniscus

$P_0 + \frac{4\sigma}{a} - P_0 - \frac{4\sigma}{b} = \frac{4\sigma}{r}$

$\Rightarrow \quad \frac{1}{r} = \frac{1}{a} - \frac{1}{b}$

$\therefore \quad r = \frac{ab}{b-a}$



4.(25) $V = 100 \times \frac{4}{3}\pi r^3 = 100 \times \frac{4}{3}\pi \times \frac{3}{40\pi} \times 10^{-15} = 10^{-14} m^3$

Vol. of oleic layer $= 10^{-16} m^3$

At $= 10^{-16}$

$4 \times 10^{-4} t = 10^{-16}$

$t = 25 \times 10^{-14} m$

5.(3) Applying Bernoulli's equation,

$P_0 + \frac{mg}{A} + \rho gh + \frac{1}{2}\rho V_1^2 = P_0 + \frac{1}{2}\rho V_2^2$

$\therefore \frac{24 \times 10}{0.4} + 1000(10)(0.4) = \frac{1000}{2} V_2^2$ (Since, $V_1 \approx 0$)

$\therefore V_2^2 = \frac{4600 \times 2}{1000}$ $\quad \therefore V_2 = \sqrt{9.2} \approx 3$

6.(A) Reynolds no. $= R = \frac{\rho VD}{\eta} = \frac{2\rho Vr}{\eta}$

$\rho =$ density; $v =$ density of flow

$r =$ radius; $\eta =$ coeffi. of viscosity

Q = Rate of flow $= \pi r^2 v \Rightarrow R = \frac{2\rho Q}{\eta \pi r}$

$R_1 =$ Reynold no. when $Q = 0.18 L$ / min $= \frac{2 \times 10^3 \times 0.18 \times 10^{-3}}{60 \times 10^{-3} \times 3.14 \times 0.5 \times 10^{-2}} \cong 382$

$R_2 =$ Reynold no. when $Q = 0.48 L$ / min $\cong 1019$

7.(B) Let the submarine is at depth h

$P_0 + \rho gh = 3 \times 10^5$

$\Rightarrow \quad 10^3 \times 10 \times h = 2 \times 10^5 \quad \Rightarrow \quad h = \frac{2 \times 10^5}{10^4} = 20m$

Now at h = 40 m

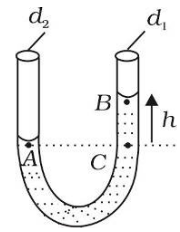
$P' = P_0 + \rho g(40)$

$= 1 \times 10^5 + 10^3 \times 10 \times 40 \quad = 5 \times 10^5$

% increase in pressure $= \frac{\Delta p}{p} \times 100$

$= \left(\frac{5 \times 10^5 - 3 \times 10^5}{3 \times 10^5}\right) \times 100 \quad = \frac{2 \times 10^5}{3 \times 10^5} \times 100 = \frac{200}{3}\%$

8.(A) $P_A = P_C$



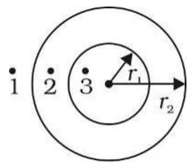
$$P_0 - \frac{4T}{d_2} = \left(P_0 - \frac{4T}{d_1}\right) + \rho gh$$

$$\frac{4T}{d_1} - \frac{4T}{d_2} = \rho gh$$

$$4 \times 7.3 \times 10^{-2} \times \frac{3}{5 \times 8 \times 10^{-3}} = 10^3 \times 10 \times h$$

$$h = 2.19\ mm$$

9.(2) $P_2 - P_1 = \dfrac{4T}{r_2}$



$$P_2 - P_0 = \frac{4T}{r_2} \qquad \text{...(i)}$$

$$P_3 - P_2 = \frac{4T}{r_1} \qquad \text{...(ii)}$$

From equation (i) & (ii) $P_3 - P_0 = \dfrac{4T}{r_2} + \dfrac{4T}{r_1}$

$$\frac{4T}{r_{eq}} = \frac{4T}{r_2} + \frac{4T}{r_1}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} = \frac{3 \times 6}{3 + 6} = 2cm$$

10.(A) $\dfrac{4\pi}{3} R^3 \times 2 = \dfrac{4\pi}{3} r^3$

$$r = 2^{1/3} R \qquad \text{... (1)}$$

$$\frac{SE_i}{SE_f} = \frac{4\pi R^2 \sigma \times 2}{4\pi r^2 \sigma} = \left(\frac{R}{r}\right)^2 (2) = \frac{2}{2^{2/3}} \qquad \text{Ratio} = 2^{1/3} : 1$$

11.(D) Velocity efflux $= \sqrt{2gh}$ (h is height of liquid)

Force $= \dfrac{dm}{dt}\sqrt{2gh} = \rho a\sqrt{2gh} \times \sqrt{2gh} = 2\rho agh$

Force of friction $= \mu \times A \times h \times g \times \rho$

$$\rho\mu Ahg = 2\rho agh \quad \Rightarrow \quad \mu = \frac{2a}{A}$$

12.(A) Since conditions are isothermal and bubbles are in vacuum.

$$P_1V_1 + P_2V_2 = P_fV_f \quad \text{and} \quad P_1 = \frac{4\sigma}{r_1} \quad P_2 = \frac{4\sigma}{r_2} \quad P_f = \frac{4\sigma}{r}$$

$$V_1 = \frac{4}{3}\pi r_1^3 \quad V_2 = \frac{4}{3}\pi r_2^3 \quad V_f = \frac{4}{3}\pi r_f^3$$

$$\therefore \quad \frac{4}{3} \times 4\sigma r_1^2 \pi + \frac{4}{3}\pi 4\sigma r_2^2 = \frac{4}{3}\pi 4\sigma r_f^2 \quad \Rightarrow \quad r_1^2 + r_2^2 = r_f^2$$

13.(A) $R = 0.2 mm$

$d_\omega = 1000 kg / m^3$

$g = 10 m / s^2$

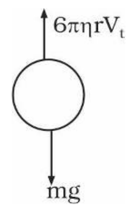
$\eta = 1.8 \times 10^{-5} Ns / m^2$

Buoyancy neglected

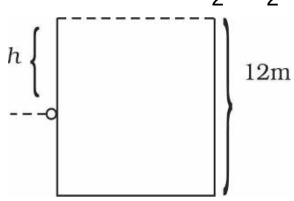
$mg = 6\pi\eta r v_t$

$d_w \frac{4}{3}\pi r^3 \times g = 6\pi\eta r v_t$

$v_t = 4.94 m / s$



14.(6) For maximum Range $h = \frac{H}{2} = \frac{12}{2} = 6$m



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1.(AC) Is correct because if $a = \frac{g}{\sqrt{2}}$, contains is in free fall

If $a = \frac{g}{2}$

$l_{12} = \sqrt{2}d$

$mg \sin\theta + (P_2 - P_1)A = ma$

$\Rightarrow \quad \rho\sqrt{2}dA \times \frac{g}{\sqrt{2}} + (P_2 - P_1)A = \rho\sqrt{2}d \times A \times \frac{g}{2}$

$\Rightarrow \quad (P_2 - P_1) = \rho g d\left(\frac{1}{\sqrt{2}} - 1\right) \quad \Rightarrow \quad \frac{(P_1 - P_2)}{\rho g d} = \frac{(\sqrt{2} - 1)}{\sqrt{2}}$

2.(0.3) **3.(10)**

For the tube to move downwards

$W = B$

$mg = \rho_{water}(v_{air} + v_{glass})g$

$5 = 1 \times \left(v_{air} + \frac{5}{2.5}\right) \Rightarrow \boxed{v_{air} = 3cc}$

$\Rightarrow \quad \Delta v = v - v \quad \Rightarrow \boxed{\Delta v = 0.3cc}$

The pressure inside the bottle will vary with depth. But since the trapped air is close to the top of the bottle, we can assume that the initial and final pressure of the trapped air will be P_0 and $P_0 + \Delta p$ respectively.

$P_0 v_0 = P_f v_f \quad \Rightarrow \quad (10^5)(3.3 \times 10^{-6}) = P_f(3.0 \times 10^{-6})$

$\Rightarrow \quad P_f = 1.1 \times 10^5 Pa \quad \Rightarrow \quad \Delta p = P_f - P_0 = 10 \times 10^3 Pa$.

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Liquids	Class - XI | Physics

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1.(363) $\rho = 800\ g\ /\ m^3$

$P_1 - P_2 = 4100\ \text{N/m}^2$

$P_1 + \rho gh + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$

$4100 + 800 \times 10 \times (1) = \frac{1}{2}\rho(3v_1^2)$

$12{,}100 = 400 \times 3 \times v_1^2$

$v_1^2 = \frac{121 \times 3}{4 \times 3 \times 3} = \frac{363}{36}$; $\quad v_1 = \frac{\sqrt{363}}{6}$

2.(C) $V_t = \frac{2(\sigma - \rho)R^2}{9\eta}$; $\quad v_t \propto R^2$

3.(100) $F = \eta A \frac{dV}{dZ} \quad \Rightarrow \quad \frac{F}{A} = \eta\left(\frac{V-0}{Z-0}\right) \Rightarrow 10^{-3} = 10^{-2}\left(\frac{10}{Z}\right) \quad \Rightarrow \quad Z = 100m$

4.(C) Direct formula – based

5.(B) $Volume = \frac{mass}{density} = \frac{m}{d_1}$

$\therefore F_V = W - F_B = mg - d_2\left(\frac{m}{d_1}\right)g = mg\left(1 - \frac{d_2}{d_1}\right)$

6.(300) $A = 0.5m^2, \quad a = 1cm^2$

$F = 250, \quad h = 40cm$

$\Delta P = \frac{1}{2}\rho V^2 \quad \Rightarrow \frac{250}{0.5} + \rho gh = \frac{1}{2}\rho V^2$

$\Rightarrow 500 + 1000 \times 10 \times 0.4 = \frac{1}{2} \times 1000 V^2$

$4.5 \times 2 = V^2$

$V = 3m\ /\ s$; $\quad V = 300cm\ /\ s$

7.(B) Time taken by ball above surface of

Lake $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 4.9}{9.8}} = 1\,\text{sec}$.

Velocity of ball when striking surface $v = \sqrt{2gh}$

$v = 9.8m\ /\ s$

Depth of lake $= v \times (t - 1) = 9.8 \times 3 = 29.4\,m$

8.(A) Volume will remain same

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = 64 \times \frac{4}{3} \times \pi r^3$$

$$\left(4\pi r^2\right) \times S - \left(4\pi R^2\right) \times S \Rightarrow r = \frac{R}{4}$$

$$\left(4\pi R^2\right)\left[\frac{64}{16} - 1\right] \times S = 4\pi \times \left(1 \times 10^{-2}\right)^2 \times 3 \times 0.075 = 2.8 \times 10^{-4} J$$

9.(D) $\eta = 1.8 \times 10^{-5} Nsm^{-2}$

$mg = 6\pi\eta r v_T$

(when F_B is negligible)

$$10^3 \times \left(10^{-6}\right)^2 \times \frac{4}{3} \times 10 \times \pi = 6 \times 1 \times \pi \times 1.8 \times 10^{-5} \times 10^{-6} \times V_T$$

$$10^3 \times 10^{-12} \times \frac{4}{3} \times 10 = 6 \times 1.8 \times 10^{-5} \times V_T$$

$$\frac{10^{-2}}{3} = 27 \times V_T$$

$$V_T = \frac{10^{-2}}{3 \times 27} = 123.45 \times 10^{-6} m / s$$

10.(24) $p_1 - p_2 = 4500 pa$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$4500 = \frac{1}{2} \times 750 \times \left(v_2^2 - v_1^2\right)$$

$\left(v_2^2 - v_1^2 = 12\right)$ $\qquad A_1 v_1 = A_2 v_2$

$4v_1^2 - v_1^2 = 12$ $\qquad A_1 v_1 = \frac{A_1}{2} \times v_2$

$3v_1^2 = 12$ $\qquad 2v_1 = v_2$

$\left(v_1 = 2m / s\right)$ $\qquad \boxed{4v_1^2 = v_2^2}$

$\left(v_2 = 4m / s\right)$ $\qquad 4 \times 4 = v_2^2$ $\qquad v_2 = 4m / s$

$A_1 v_1 =$ rate of flow $= 1.2 \times 10^{-2} \times 2$

$= 2.4 \times 10^{-2} m^3 / s$ $\qquad = 24 \times 10^{-3} m^3 / s$

11.(20) $r = 0.1 \times 10^{-3} m = 10^{-4} m$

$\rho_0 = 10^4 kgm^{-3}$

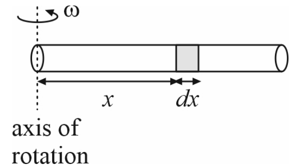
$$\sqrt{2gh} = V_T\ ; \qquad \sqrt{2gh} = \frac{2(\rho_0 - \rho_\ell) \times r^2 g}{9\eta}$$

$$2gh = \frac{4}{81}\left(\frac{(\rho_0 - \rho_\ell)}{\eta}\right)^2 r^4 g^2$$

$$h = \frac{2}{81}\left(\frac{(10^4 - 10^3)}{10^{-5}}\right)^2 (10^{-4})^4 \times 10\ ; \qquad h = 20m$$

12. Given, $l = 50cm$

$\text{mass}(m) = 250gm$



mass per unit length $= \frac{m}{l}$

Take element of 'dx' from axis of rotation, having

$$\text{mass } dm = \frac{m}{l}dx$$

Small force (dF) exerted by liquid

$$dF = dm \cdot x \cdot \omega^2 = \frac{M}{l}dx \cdot x \cdot \omega^2$$

For net force at other end we have to solve for $x = 0$ to $x = l$

$$\int dF = \int_0^x \frac{M}{l}\omega^2 x dx = \frac{M}{l}\omega^2\left[\frac{x^2}{2}\right]_0^l$$

$$F = \frac{M}{l}\omega^2 \times \frac{l^2}{2}; \quad F = \frac{1}{2}ml\omega^2 \Rightarrow \omega^2 = \frac{2F}{Ml}$$

$\Rightarrow \qquad \omega = \sqrt{\frac{2F}{Ml}}$, given that $\omega = x\sqrt{F}$

So $\qquad x = \sqrt{\frac{2}{Ml}} = \sqrt{\frac{2}{250 \times 10^{-3} \times 50 \times 10^{-2}}}$; $\qquad x = 4$ Ans

13.(A) Here $F_B + T \times 2\pi r = mg$; $\qquad \frac{2}{3}\pi r^3 \times \sigma g + T \times 2\pi r = \frac{4}{3}\pi r^3 \times \rho \times g$

$$\frac{2}{3}\pi r^3 \times \sigma g + T \times 2\pi r = \frac{4}{3}\pi r^3 \times \rho g; \; T \times 2\pi r = \frac{2}{3}\pi r^3 g(2\rho - \sigma)$$

$$T = \frac{gr^2}{3}(2\rho - \sigma); \qquad r = \sqrt{\frac{3T}{(2\rho - \sigma)g}}; \; r = \frac{15}{\sqrt{2\rho - \sigma}}$$

14.(C) $\frac{4\pi}{3}(1cm)^3 = \frac{4\pi}{3}(r)^3 \times 729$

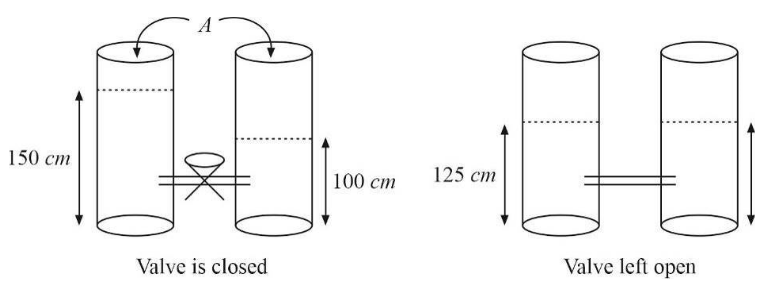
$9r = 1cm$; $\qquad r = \frac{1}{9}cm$

Gain in surface energy $= \left[4\pi r^2 \times 729 - 4\pi(1cm)^2\right]\sigma$

$$= 4\pi\left(\frac{729}{81} - 1\right)\sigma\, cm^2 \qquad = 4\pi \times 8 \times 10^{-4} \times \frac{75 \times 10^{-5} J}{10^{-2}}$$

$$= 32 \times 3.14 \times 75 \times 10^{-7} J \quad = 7.5 \times 10^{-4} J$$

15.(B)

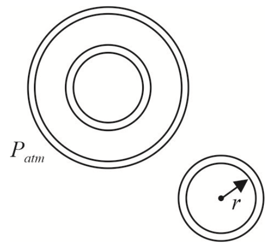


$W_{mg} = mgh$

$= (\rho_w \times A \times 0.25) \times 10 \times 0.25$

$= 10^3 \times 16 \times 10^{-4} \times \frac{1}{4} \times 10 \times \frac{1}{4} \qquad = 1 J$

16.(2) $P_1 = P_{atm} + \frac{4S}{6} + \frac{4S}{3}$



$P_2 = P_{atm} + \frac{4S}{r}$

$P_1 = P_2$

$\Rightarrow \quad \frac{1}{6} + \frac{1}{3} = \frac{1}{r}; \qquad r = 2cm$

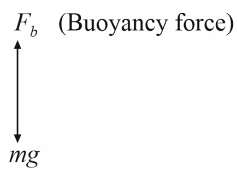
17.(11) Using $v = \frac{2\rho r^2 g}{8\eta}$

We get $\eta = \frac{9.8}{9} = 1.088 \; N\,m^{-2}\sec \qquad = 10.88 \;$ poise

18.(C)

19.(A)

20.(25) For ball



$\downarrow \; \rho_l \rightarrow$ density of liquid $= 1.3g / cc$

$\rho_b \rightarrow$ density of ball $= 8g / cc$

Viscous Force $(F_v) = mg - F_b$

$F_v = mg - v\rho_l g$

$= v\rho_b g - v\rho_l g = v\rho_b g\left[1 - \frac{\rho_l}{\rho_b}\right] \qquad = mg\left[1 - \frac{\rho_l}{\rho_b}\right]$

$F_v = 0.3 \times 10^{-3} \times 10\left[1 - \frac{1.3}{8}\right] \qquad = 25 \times 10^{-4} N$

Value of $x = 25$

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1.(A) $\Delta V = V_0 \gamma \Delta T \quad = a^3(3\alpha\Delta T)$

$(\because \quad \gamma = 3\alpha)$

2.(D) $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta} \quad \Rightarrow \quad Y = \frac{9K\eta}{\eta + 3K}; K = \frac{Y\eta}{9\eta - 3Y}$ and $\eta = \frac{3Ky}{9K - y}$

$\therefore$ Option (D) is correct

3.(2) $\frac{\text{stress}}{\text{strain}} = Y \quad \Rightarrow \quad \frac{F / A}{\Delta L / L} = Y \quad \Rightarrow \quad \Delta L = \frac{FL}{YA} = \frac{FL}{Y\left(\frac{\pi d^2}{4}\right)}$

So, $\Delta L \propto \frac{L}{d^2} \quad \Rightarrow \quad \Delta L_2 = \frac{1}{2}\Delta L_1 = 0.02 \text{ m} = 2 \text{ cm}$

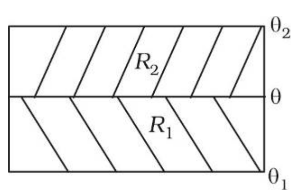
4.(D) Thermal stress is developed when both the ends of the rod are fixed and the rod is heated or cooled.

5.(C) $K(\text{Bulk modulus}) = \frac{\Delta P}{-\frac{\Delta V}{V}}$

$\frac{m}{v} = \rho \quad \Rightarrow \quad m = \rho v \quad \Rightarrow \quad 0 = \rho dv + v d\rho \quad \Rightarrow \quad -\rho dv = v d\rho \quad \Rightarrow \quad \frac{dv}{v} = \frac{-d\rho}{\rho}$

Now, $k = \frac{\Delta P}{\frac{d\rho}{\rho}} \quad \Rightarrow \quad \frac{d\rho}{\rho} = \frac{P}{K} \quad \Rightarrow \quad d\rho = \frac{P\rho}{K}$

6.(C) $\frac{\theta_1 - \theta}{R_1} = \frac{\theta - \theta_2}{R_2}$



$R_2\theta_1 - R_2\theta = R_1\theta - R_1\theta_2$

$R_2\theta_1 + R_1\theta_2 = R_1\theta + R_2\theta$

$\frac{R_2\theta_1 + R_1\theta_2}{R_1 + R_2} = \theta$

7.(B) $\frac{K(l_1 - l_0) = T_1}{K(l_2 - l_0) = T_2}; \quad \frac{T_1}{T_2} = \frac{(l_1 - l_0)}{(l_2 - l_0)}$

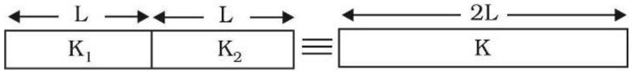
$T_1(l_2 - l_0) = T_2(l_1 - l_0)$

$T_1 \times l_2 - T_1 \times l_0 = T_2 l_1 - T_2 l_0$

8.(C) Hydraulic stress $P_0 + \rho g h \approx 19.6 \times 10^6 N/m^2$

$\frac{\text{hydraulic stress}}{\text{hydraulic strain}} = \frac{19.6 \times 10^6}{1.36 \times 10^{-2}} = 1.44 \times 10^9 N/m^2$

9.(D)



$R_1 + R_2 = R_{eq}$

$\Rightarrow \quad \dfrac{L}{K_1 A} + \dfrac{L}{K_2 A} = \dfrac{2L}{KA} \qquad \left[\because R_{Th} = \dfrac{L}{KA}\right]$

$\Rightarrow \quad \dfrac{1}{K_1} + \dfrac{1}{K_2} = \dfrac{2}{K} \quad \Rightarrow \quad \dfrac{2}{K} = \dfrac{K_1 + K_2}{K_1 K_2} \quad \Rightarrow \quad K = \dfrac{2K_1 K_2}{K_1 + K_2}$

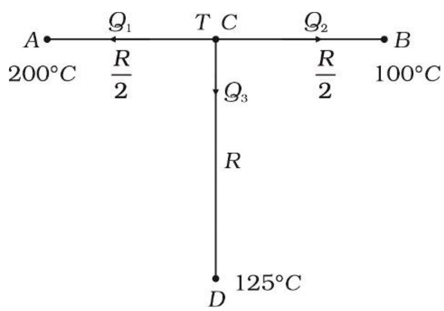
10.(32) Given $2\Delta\ell_A = \Delta\ell_B$

$\dfrac{2f}{A_a \gamma}\ell_A = \dfrac{F\ell_B}{16 A_a \gamma}; \qquad \dfrac{\ell_A}{\ell_B} = \dfrac{1}{32}$

11.(A) $\alpha_A > \alpha_B$

So length of A should decrease more and since both A and B were welded together bimetallic strip will bent towards left.

12.(2)



$Q_1 + Q_2 + Q_3 = 0$

$\Rightarrow \quad \dfrac{T-200}{R/2} + \dfrac{T-125}{R} + \dfrac{T-100}{R/2} = 0 \quad \Rightarrow \quad 2T - 400 + T - 125 + 2T - 200 = 0$

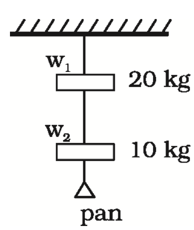
$\Rightarrow \quad 5T - 725 = 0 \quad \Rightarrow \quad T = 145°C$

Heat Constant = $\dfrac{\Delta V}{R_{thermal}} = \dfrac{145-125}{10} = \dfrac{20}{10} = 2W$

13.(C) $\because$ Loss of gravitational potential energy $=$ mass $\times$ specific heat capacity $\times$ change in temperature

$mgh = ms\Delta T \Rightarrow \Delta T = \dfrac{9.8 \times 63}{4200} = 0.147$

14.(40) Stress on w_1 wire



$\sigma_1 = \dfrac{F}{A} = \dfrac{(20+10+m)g}{8 \times 10^{-7}}$

$1.28 \times 10^9 = \dfrac{(10+20+m) \times 10}{8 \times 10^{-7}}$

$1.25 \times 10^9 \times 8 \times 10^{-7} = (10+m)10$

$1.25 \times 10^9 \times 8 \times 10^{-7} = (30+m)10$

$1.25 \times 8 \times 10 = (30+m)$; $100 - 30 = m$; $m = 70$

Stress on w_2 wire $\qquad 1.25 \times 10^9 = \dfrac{(10+m)g}{4 \times 10^{-7}}$

$1.28 \times 4 \times 10^9 \times 10^{-7} = (80+m) \times 10$; $\qquad m = 40kg$

So maximum mass of pan without breaking the wires is 40 Rs.

15.(B) $MS_x(6) + MS_y(-4) = 0 \longrightarrow 3S_x = 2S_y$

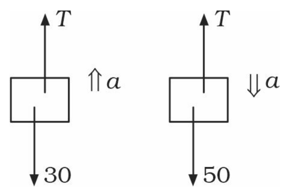
$MS_y 6 + MS_z(-4) = 0 \longrightarrow 3S_y = 2S_z \to 9S_x = 4S_z$

$MS_x(T-10) + MS_z(T-30) \longrightarrow \frac{4}{9}(T-10) = -T + 30$

$4T - 40 = -9T + 270$

$13T = 310$; $\quad T = \frac{310}{13} = 23.84°C$

16.(C) $\frac{50-T}{5} = a = \frac{T-30}{3}$



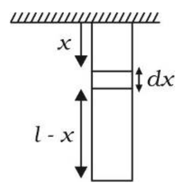
$150 - 3T = 5T - 150$

$T = \frac{300}{8}$; $\quad$ Stress $= \frac{T}{A}$

$\frac{24}{\pi} \times 10^2 = \frac{300}{8 \times \pi \times r^2}$

$r^2 = \frac{1}{8^2}$; $\quad r = \frac{1}{8} m = 12.5\ cm$

17.(B) $dl = \frac{Fl}{Ay} = \frac{Tdx}{Ay}$



$\Delta L = \int dl = \int_0^l \frac{w}{l} \frac{(l-x)}{Ay} dx$

$\Delta L = \frac{w}{lAy}\left[lx - \frac{x^2}{2}\right]_0^l = \frac{wl}{2Ay}$

$= \frac{10 \times 0.2}{2 \times 0.01 \times 2 \times 10^{11}} = 5 \times 10^{-10} m$

18.(500) We know,

$B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$ $\quad \Rightarrow \quad B = -\frac{\rho g h}{\frac{\Delta V}{V}}$

$\Rightarrow \quad h = -\frac{B}{\rho g}\left(\frac{\Delta V}{V}\right)$ $\quad$ Given, $\frac{\Delta V}{V} = -\left(\frac{0.5}{100}\right)$

$\Rightarrow \quad h = \frac{9.8 \times 10^8 \times 0.5}{10^3 \times 9.8 \times 100}$ $\quad B = 9.8 \times 10^8\ N - m^2$

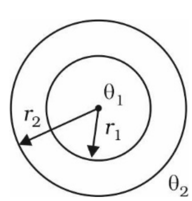
$\Rightarrow \quad h = 500$ $\quad \rho = 10^3\ kg - m^{-3}$

19.(8) $\Delta l = l\alpha\Delta T$

tension $(F) = \frac{AY\Delta l}{l} = \frac{AYl\alpha\Delta T}{l}$

solving $F = 8 \times 10^5$, $x = 8$

20.(D) $H = \frac{\theta_2 - \theta_1}{R}$



$R = \frac{r_2 - r_1}{4\pi k\,(r_1 r_2)}$

$H = \frac{4\pi k\,(r_1 r_2)}{(r_2 - r_1)}(\theta_2 - \theta_1)$

21.(A) $\frac{\Delta l}{l}=\frac{Mg/4}{AY}=\frac{5\times10^4\times9.8}{\pi(1-0.25)\times2\times10^{11}\times4}=2.60\times10^{-7}$

22.(B) $Q=I^2Rt$ ...(1)

also $Q=mS\Delta T+mL$ ...(2)

mass of ice = ρAH

solving $t=35.3$ sec

23.(A) $\frac{l}{AY}=\frac{\Delta l}{T}$; $\frac{l}{AY}=\frac{l_1-l}{T_1}$

Also, $\frac{l}{AY}=\frac{l_2-l}{T_2}$ $\Rightarrow$ $\frac{l_1-l}{T_1}=\frac{l_2-l}{T_2}$

$\Rightarrow$ $l_1T_2-lT_2=l_2T_1-lT_1$ $\Rightarrow$ $l_1T_2-l_2T_1=l\left(T_2-T_1\right)$ $\therefore$ $l=\frac{l_1T_2-l_2T_1}{T_2-T_1}$

24.(D) Constant rate = R

For A; $R\times3=m.S_A.120$

For B: $R\times6=m.S_B.90$ $\Rightarrow$ $\frac{S_A}{S_B}=\frac{3}{8}$

25.(A) Equivalent spring constant for wire 1 $=k_1=\frac{Y_1A}{L}$

Equivalent spring constant for wire 2 $=k_2=\frac{Y_2A}{L}$

So, equivalent spring constant for combination $=\frac{k_1k_2}{k_1+k_2}=\frac{Y_1Y_2A}{\left(Y_1+Y_2\right)L}=\frac{Y_{eq.}A}{2L}$

$\Rightarrow$ $Y_{eq}=\frac{2Y_1Y_2}{Y_1+Y_2}$

26.(A) Form Newton law of cooling

$\frac{\Delta T}{t}=k\,(T_{mean}-T_{sur})$

$\frac{61-59}{4}=k\left(\frac{61+59}{2}-30\right)$

$\frac{1}{2}=k(60-30)$; $k=\frac{1}{60}$

$\frac{51-49}{t}=k\left(\frac{51+49}{2}-30\right)$; $\frac{2}{t}=\frac{1}{60}(50-30)$ $\Rightarrow$ $t=6$

27.(20) $\frac{F}{A}=y\frac{\Delta L}{L}$ $\Rightarrow$ $F=\frac{yA}{L}\Delta L$

Comparing with $F=kx\leftarrow$ magnitude form

$k=\frac{yA}{L}$

$-\Delta P.E=\Delta K.E$ (as stone is released from catapult)

$\frac{1}{2}kx^2=\frac{1}{2}mv^2$; $\frac{Ay}{L}(\Delta L)^2=mv^2$ $\Rightarrow$ $\frac{10^{-6}\times0.5\times10^9}{0.1}\times(0.04)^2=\frac{20}{1000}v^2$

$v^2=0.5\times\frac{10^4}{2}\times16\times10^{-4}\times10^2=4\times10^2$; $v=2\times10=20$ m/s

28.(5) Strain in $1m$ track $= \alpha \ \Delta T = 10^{-5} \times 10 = 10^{-4}$

Energy in $1m$ track $= \frac{1}{2} \times Y \times (strain)^2 \times vol$

$= \frac{1}{2} \times 10^{11} \times \left(10^{-4}\right)^2 \times \left(10^{-2}\right) \times 1 \quad = 5J$

29.(57) $\ln\left(\frac{T - T_0}{T_i - T_0}\right) = kt$

$\ln\left(\frac{T-25}{65-25}\right) = \ln\left(\frac{65-25}{75-25}\right)$

$\frac{T-25}{40} = \frac{40}{50}$

$T = 25 + 32 = 57°C$

30.(D) In case 1

$\Delta L_1 = L - l_1$, where L is original length

$Y = \frac{T_1 L}{A\,\Delta L_1} = \frac{T_1 L}{A\left(L - l_1\right)} \quad \ldots$(i)

Where A is cross – sectional area.

Similarly in case 2

$Y = \frac{T_2 L}{A\left(L - l_2\right)} \quad \ldots$(ii)

From (i) & (ii)

$\frac{T_1}{L - l_1} = \frac{T_2}{L - l_2}$ or $L = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$

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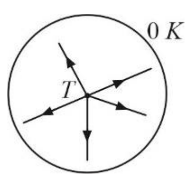
1.(9) $T_s = 0\ K$

$T_i = 200\ K$, $e = 1$

$-Ms\frac{dT}{dt} = \frac{dQ}{dt} = \sigma e A T^4$



$-\frac{dT}{dt} = \frac{\sigma A T^4}{Ms}$

$\frac{\sigma A}{Ms}\int_{t_i}^{t_f} dt = -\int_{T_i}^{T_f} \frac{dT}{T^4}$

$\frac{\sigma A}{Ms}(t_f - t_i) = +\frac{1}{3}\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right)$

$$\frac{\frac{\sigma A}{Ms}(t_1 - 0) = \frac{1}{3}\left(\frac{1}{100^3} - \frac{1}{(200)^3}\right)}{\frac{\sigma A}{Ms}(t_2 - 0) = \frac{1}{3}\left(\frac{1}{(50)^3} - \frac{1}{(200)^3}\right)}$$

$$\frac{t_1}{t_2} = \frac{\frac{(200)^3 - (100)^3}{(100)^3(200)^3}}{\frac{(200)^3 - (50)^3}{(50)^3(200)^3}} = \frac{\frac{4^3 - 2^3}{2^3}}{4^3 - 1^3} = \frac{56}{8 \times 63} = \frac{1}{9}$$

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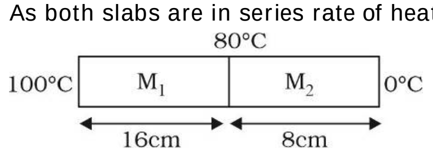
1.(C) $P = -B\dfrac{\Delta V}{V}$

$= 3\times10^{10}\times\dfrac{2}{100}$

$= 6\times10^{8}\ Nm^{-2}$

2.(B) As both slabs are in series rate of heat flow will be same in both



$\dfrac{\Delta H}{\Delta t} = \dfrac{K_1A(100-80)}{16} = \dfrac{KA(80-0)}{8}$

$K_1\times\dfrac{20}{2} = 80K\ ;\ K_1 = 8K$

3.(C) $\dfrac{1}{4}\left(\dfrac{1}{2}mv^2\right) = ms\Delta T$

$\Rightarrow \dfrac{1}{4}\left(\dfrac{1}{2}\times1.5\times60^2\right) = 0.1\times420\times\Delta T$

$\Rightarrow \dfrac{1}{4}\left(\dfrac{3}{4}\times3600\right) = 42\,\Delta T \qquad \Rightarrow\ \Delta T = \dfrac{3\times3600}{16\times42} = 16.07°C$

4.(16) $(T_f - T_0) = (T_L - T_0)e^{-kt}$

From $60\to40$

$40 = 60e^{-k\times6} \qquad \ldots(i)$

$\dfrac{2}{3} = e^{-k6} - k\times6 = \ln\dfrac{2}{3} \Rightarrow 6k = \ln1.5$

From $60\to20$; $\qquad 20 = 60e^{-kt2}$

$\dfrac{1}{3} = e^{-kt2} \Rightarrow \ln3 = t_2k$

$t_2 = \dfrac{\ln3}{\ln1.5}\times6 \approx 16\ \text{min}$

5.(31) Heat rejected per minute

$= 50\times(540) + 50\times1\times(100-20)$

$= 31\times10^3 cal$; 31

6.(D) Volume = $V_0 = \left(\dfrac{\text{surface area}}{6}\right)^{3/2}$

$V_0 = 8\times10^6\ \text{cm}^3 \qquad \therefore \qquad \Delta V = V_0(3\alpha)\Delta T = 1.2\times10^5\ \text{cm}^3$

7.(C) Maximum heat lost by copper block = $MS(T-0)$

Latent heat required to melt m mass of ice = mL

Equating both $5 \times 0.39 \times 500 = m \times 335$

$m = 2.9$ kg

8.(25) Slope $= \dfrac{20}{10^{-10}}$

$\therefore$ Stress at 5×10^{-4} is $\dfrac{20 \times 5 \times 10^{-4}}{10^{-10}}$

$= \dfrac{10^{-2}}{10^{-10}} = 10^{8}\ \text{N/m}^2$

Energy density $= \dfrac{1}{2}$ stress × strain

$= \dfrac{1}{2} \times 10^{8} \times 5 \times 10^{-4} \qquad = 2.5 \times 10^{4}$

$= 25000 \qquad = 25\ \text{kJ/m}^3$

9.(6) Elongation is due to its own mass.

$\Delta \ell \propto g$; $10^{-4} \propto 10$

$6 \times 10^{-5} \propto g'$

$\dfrac{10^{-4}}{6 \times 10^{-5}} = \dfrac{10}{g'}$; $\quad g' = 6$

10.(42) $\dfrac{\Delta m}{\Delta t} C \Delta T = \left(\dfrac{dm}{dt}\right) \times H_{\text{combustion}}$

$2 \times 10^{3} \times 4.2 \times 40 = \dfrac{dm}{dt} \times 8 \times 10^{3}$

$\Rightarrow \dfrac{dm}{dt} = 42 g\ \text{min}^{-1}$

11.(B) $Q = m S \Delta T + mL = 0.4\left(\dfrac{1}{2} m v^2\right) \quad \Rightarrow \quad v^2 = \left(\dfrac{S \Delta T + L}{0.2}\right)$

$v^2 = \dfrac{125 \times 200 + 2.5 \times 10^{4}}{0.2} \qquad$ Or $\qquad v = 500 m/s$

12.(C) $\Delta L = \dfrac{WL}{YA}$

$L_1 - L = \dfrac{(1g)L}{YA}$ . . .(i)

$L_2 - L = \dfrac{(2g)L}{YA}$ . . .(ii)

Divide $\dfrac{L_1 - L}{L_2 - L} = \dfrac{1}{2} \Rightarrow 2L_1 - 2L = L_2 - L$

$L = 2L_1 - L_2$

13.(21) Equivalent resistance of the combination

$\dfrac{4}{KA} + \dfrac{2.5}{(2K)A} = \dfrac{6.5}{K_{eq} A} \quad \Rightarrow \quad \dfrac{6.5}{K_{eq}} = \dfrac{16}{4K} + \dfrac{5}{4K}$

$K_{eq} = \dfrac{6.5 \times 4}{21} K = \dfrac{26}{21} K = \left(1 + \dfrac{5}{21}\right) K$; $\qquad \alpha = 21$

14.(D) $d = d_0(1 + \alpha\Delta T)$

$6.241 = 6.230[1 + 1.4 \times 10^{-5}(T - 27)]$

$\frac{6.241}{6.230} = 1 + (1.4 \times 10^{-5})(T - 27)$

$\frac{1.00176 - 1}{1.4 \times 10^{-5}} = T - 27$; $T = 152.7°C$

15.(5) $\Delta\ell_1 = \frac{F\ell}{AY} = \frac{F\ell}{\pi r^2 Y} = 5\,cm$; $\Delta\ell_2 = \frac{4F4\ell}{\pi 16 r^2 Y} = \frac{F\ell}{\pi r^2 Y} = 5\,cm$

16.(60) $\ell_B(1 + \alpha_B\Delta T)^* - \ell_i(1 + \alpha_i\Delta T) \quad = \ell_B - \ell_i$

$\ell_B\alpha_B\Delta T = \ell_i\alpha_i\Delta T$; $\ell_B\alpha_B = \ell_i\alpha_i$

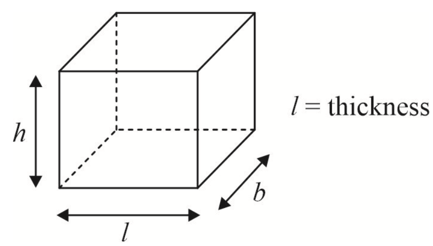
$\ell_i = \frac{\ell_B\alpha_B}{\alpha_i} = \frac{40 \times 1.8 \times 10^{-5}}{1.2 \times 10^{-5}} = 60\,cm$

17.(90) Heat absorbed by x gram ice = Heat released by 300 g water

$xL = mS\Delta\theta$; $x \times 3.5 \times 10^5 = 300 \times 4200 \times (25 - 0)$

$x = \frac{3 \times 42 \times 25}{35} = 90\,gram$

18.(B) Heat energy is being tranferred to the ice cube through the Box.
We assume that the ice cube first fits in the box.



So thermal current $= \frac{dm}{dt}L$

$2.\frac{K\,l\,b\;\Delta\theta}{t} + \frac{2K\,bh\;\Delta\theta}{t} + \frac{2K\,lh\,\Delta\theta}{t} = \frac{dm}{dt}L$

$\frac{2K\,\Delta\theta}{t}\left(lb + bh + lh\right) = \frac{dm}{dt}L$

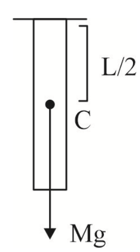
Solving $\frac{dm}{dt} = 61 \times 10^{-5}\,kg\,/\,\sec$

19.(A) (Breaking stress remains same)

$\frac{F_1}{a_1} = \frac{F_2}{a_2}$

Property of material $\boxed{a_2 = 2.5a_1}$

20.(25) The extension of center of mass assuming whole weight to be acting on it strain $= \frac{\text{stress}}{Y}$

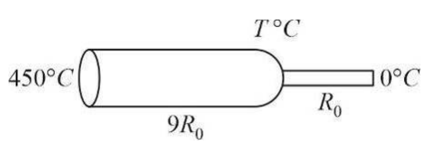


$\Delta y(\text{Extension}) = \frac{Mg\,L}{AY\,2}$

Putting the values $\Delta y = 25 \times 10^{-9}m = x \times 10^{-9} \Rightarrow x = 25$

21.(C) Thermal resistance, $R = \frac{L}{KA}$

Let $R_1 = \frac{L_1}{K_1A_1} = 9R_0$

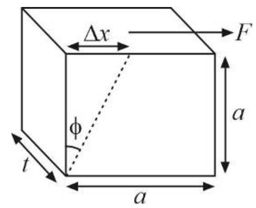


Then $R_2 = \frac{L_2}{K_2A_2} = R_0$

At steady state,

$\frac{450 - T}{9R_0} = \frac{T - 0}{R_0} \quad \Rightarrow \quad 450 = 10T \quad \Rightarrow \quad T = 45°$

22.(48) $\frac{F}{at} = G \times \phi$



$\frac{F}{at} = G \times \frac{\Delta x}{a}$

$\frac{F}{Gt} = \Delta x$

$\frac{18 \times 10^4}{25 \times 10^9 \times 15 \times 10^{-2}} = \Delta x$

$\frac{6}{125} \times 10^{-3} = \Delta x$

$48 \times 10^{-6} = \Delta x$

23.(D) $k_1 = \frac{Y_1 A}{l_1} = \frac{2 \times 10^{11} \times \pi \times \left(1.4 \times 10^{-3}\right)^2}{3.2}$

$k_2 = \frac{Y_2 A}{l_2} = \frac{1.1 \times 10^{11} \times \pi \times \left(1.4 \times 10^{-3}\right)^2}{4.4}$

$\frac{1}{k_{eqv.}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{3.85 \times 10^5} + \frac{1}{1.5386 \times 10^5}$

$= 259.74 \times 10^{-8} + 649.94 \times 10^{-8}$

$k_{eq} = 1.099 \times 10^{-3} \times 10^8$

$F = k_{eqv.} x_0 = 1.099 \times 10^5 \times 1.4 \times 10^{-3} = 1.538 \times 10^2 = 153.8 \approx 154$

24.(C) Strain $= \frac{\Delta L}{L} = 1$

$\frac{F}{A} = Y \times 1 \Rightarrow F = YA$

$F = 2 \times 10^{11} \times 10^{-4}$

$F = 2 \times 10^7 N$

25.(30) Stress $= \frac{F}{A} = Y \times$ Strain

Strain $= \frac{F}{AY}$

$= \frac{mg + \frac{mV^2}{r}}{AY} = \frac{20 + \frac{2(5)^2}{0.5}}{4 \times 10^{-6} \times 10^{11}} \qquad = \frac{20 + 100}{4 \times 10^5} = 30 \times 10^{-5}$

26.(A) Young's modulus depends on material of body.

27.(50) $\bullet\!\!\longrightarrow\!\leftarrow \boxed{10kg} \rightarrow = \frac{mv^2}{r}$

Breaking stress $= \frac{T_{\max}}{Area} \Rightarrow T_{\max} = 5 \times 10^8 \times 10^{-4}$

$= 5 \times 10^4 N$

At breaking stress linear velocity will be maximum

$T_{\max} = \frac{mv^2}{R}$

$v = \sqrt{\frac{TR}{m}} = \sqrt{\frac{5 \times 10^4 \times 0.5}{10}}$; $\qquad v = 50 \, m/s$

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Solutions of Archive - JEE Main & Advanced

Gaseous State and Thermodynamics	Class - XI | Physics

JEE Main 2021

1.(C)

(a)	isothermal	(ii)	temperature constant
(b)	isochoric	(iii)	volume constant
(c)	adiabatic	(iv)	heat content is constant
(d)	isobaric	(i)	pressure constant

2.(A) $(\text{work})_{AB} = nRT \ln\left(\frac{V_2}{V_1}\right)$ {isothermal process}

$= nRT \ln 2$

$(\text{work})_{BC} = P\Delta V$ {isobaric process}

$= P_2(V_1 - V_2)$

$= P_2(V_1 - 2V_1)$

$= \frac{-P_1V_1}{2}$ $(\because P_2 2V_1 = P_1V_1)$

$= \frac{-nRT}{2}$

$(\text{work})_{net} = 0$ {isochoric process}

$(\text{work})_{net} = nRT \ln 2 - \frac{nRT}{2}$ $= nRT(\ln 2 - 1/2)$

3.(D) Due to the collision of the molecules with the walls, the molecules exert a force (and hence exert a pressure) on the walls. In the process, the momentum of the molecules changes since they experience an equal and opposite force.

4.(C) It is not mentioned whether 'T' is the temperature at A, B or C.

Assuming that `T' is the temperature at A,

$$P_1V_1 = RT$$

Work done in A to B,

$$W_{AB} = RT \ \ln \frac{V_B}{V_A} = RT \ \ln 2$$

Work done in B to C is zero

Work done in C to A, $W_{CA} = \frac{\Delta PV}{1-\gamma}$

$$= \frac{P_1V_1 - \left(\frac{P_1}{4}\right)(2V_1)}{1-\gamma} \qquad = -\frac{P_1V_1}{2(\gamma-1)} = -\frac{RT}{2(\gamma-1)}$$

So, net work $W_{\text{net}} = W_{AB} + W_{BC} + W_{CA}$

$$= RT\left[\ln 2 - \frac{1}{2(\gamma-1)}\right]$$

5.(400) The rms speed depends only on temperature.

$$v_{rms} = \sqrt{\frac{3RT}{M_0}} \quad \Rightarrow \quad \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{400}{300}} \qquad \Rightarrow \quad v_2 = \frac{2}{\sqrt{3}}\ v_1 = \frac{400}{\sqrt{3}}\ \text{m/s}$$

6.(D) $dU : dQ : dW = nC_v dT : nC_p dT : nRdT = C_v : C_p : R = \frac{5}{2}R : \frac{7}{2}R : R \ = 5:7:2$

7.(3600) $\frac{MV^2}{2} = C_V T = \frac{3R}{2}T$

$$\Rightarrow \qquad T = \frac{MV^2}{3R} \Rightarrow MV^2 = 4 \times 900 = x\ ;\ \ x = 3600$$

8.(B) Rotational energy obeys maxwell's distribution.

For a diatomic molecule

Rotational energy / mole = RT

Translational energy / mole $= \frac{3RT}{2}$

9.(B) $PV^{1/2} =$ Constant

$\frac{nRT}{V}V^{1/2}$ = Constant

So, $\quad \frac{T}{V^{1/2}} =$ Constant

So, $\quad \frac{T_1}{\sqrt{V_1}} = \frac{T_2}{\sqrt{V_2}}; \qquad T_2 = T_1\sqrt{\frac{V_2}{V_1}}; \qquad \frac{T_2}{T_1} = \sqrt{2}$

10.(208) $\eta = \frac{W}{Q} = \frac{1}{4} = \frac{52}{T} \qquad \Rightarrow \qquad T = 208$

11.(26) $\frac{f}{2}n_1RT_1 + \frac{f}{2}n_2RT_2 = \frac{f}{2}(n_1+n_2)RT$

$P_1V_1 + P_2V_2 = PV\ ; \qquad 2(4.5) + 3(5.5) = P(10)$

$25.5 = P(10)\ ; \quad P = 25.5 \times 10^{-1}$

Rounding off $= 26$

12.(D) $U = 3PV + 4\ ; \quad \frac{du}{dT} = \frac{3d(pv)}{dT}$

$du = nC_V dT\ ; \quad \frac{du}{dT} = nC_V$

$Pv = nRT\ ; \qquad PV = nC_V = 3nR$

$C_V = 3R\ ; \qquad \Rightarrow\ f = 6$ since $C_v = \frac{fR}{2}$

13.(Bonus) $V = KT^{2/3} = K\left(\frac{PV}{nR}\right)^{2/3} \qquad \Rightarrow \qquad V \propto P^2 \ \Rightarrow\ PV^{-1/2} = \text{constant}$

For $\quad PV^x = \text{constant}, W = \frac{nR\Delta T}{1-x}$

Here $x = -\frac{1}{2} \quad \therefore \qquad \omega = \frac{nR(90)}{(3/2)} = 60\ nR$

As number of moles is not given, so answer cannot be found.

14.(25) Given diatomic gas, $Q - \frac{Q}{5} = \Delta U = \frac{4Q}{5} = n\frac{5}{2}R\Delta T$

or $\quad Q = n\frac{25}{8}R\Delta T \quad \Rightarrow \quad$ Thus $C_{\text{process}} = \frac{25R}{8}$

Since $\quad Q = nC_{\text{process}}\Delta T$

Hence $\quad x = 25$

15.(B) $\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{30}$

$\gamma = 1 + \frac{1}{15} = \frac{16}{15}$

$\gamma = 1.07$

16.(B) $\eta = \frac{W}{Q} = 1 - \frac{T_L}{T_H}$

$\Rightarrow \quad \frac{1200J}{Q} = 1 - \frac{400}{800}$

$\therefore \quad Q = 2400J$

17.(A) Heat given = Heat taken,

$n_1F_1(T_1 - T) = n_2F_2(T - T_2)$

$\Rightarrow \quad n_1F_1T_1 - n_1F_1T = n_2F_2T - n_2F_2T_2$

$\Rightarrow \quad n_1F_1T_1 + n_2F_2T_2 = n_1F_1T + n_2F_2T$

$\Rightarrow \quad n_1F_1T_1 + n_2F_1T_2 = (n_1F_1 + n_2F_2)T$

$\therefore \quad T = \frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$

18.(C) $PV = (n_1 + n_2 + n_3)RT$

$n_1 = \frac{16}{32} = 0.5\,mole$

$n_2 = \frac{28}{28} = 1\,mole$

$n_3 = \frac{44}{44} = 1\,mole$

$\therefore \quad PV = \frac{5}{2}RT \quad$ or $\quad P = \frac{5RT}{2V}$

19.(C) Heat and work are path functions

20.(C) $f + 3 + 3 + 2 \times 2 = 10$

$C_V = \frac{10R}{2} = 5R$

$C_P = C_V + R = 6R$

$\beta = \frac{C_P}{C_V} = \frac{6R}{5R} = 1.2$

21.(C)

22.(C) $\frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$\frac{5}{2}(100 \times 2 - 200 \times 3) = -500\ J$

23.(D) As per law of equipartition of energy.

24.(D) PV^{γ} = constant

$V\gamma\Delta P + \gamma V^{\gamma-1}\Delta VP = 0$

Dividing by $V^{\gamma-1}$

$V\Delta P + \gamma\Delta VP = 0$

$\frac{\Delta P}{P} = -\gamma\frac{\Delta V}{V}$

25.(D) $S_1 = \frac{f}{2}n_1R$

$S_2 = \frac{f}{2}n_2R$

Entropy of system $= \frac{f}{2}(n_1 + n_2)R$

$= \frac{f}{2}\left(\frac{2S_1}{fR} + \frac{2S_2}{fR}\right)R = S_1 + S_2$

26.(B) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$V_{av} = \sqrt{\frac{8RT}{\pi M}}$ $\quad\therefore\quad \frac{V_{rms}}{V_{av}} = \sqrt{\frac{3\pi}{8}}$

27.(B) Mean free path, $\lambda = \frac{kT}{\sqrt{2}\pi d^2P} = \frac{1.38\times10^{-23}\times300}{\sqrt{2}\times3.14\times\left(0.3\times10^{-9}\right)^2\times1.01\times10^5} = 102\times10^{-9}m = 102nm$

d :Molecule diameter; P: Pressure; T: Temperature.

28.(-113) $\eta = 1-\frac{T_2}{T_1}$ $\Rightarrow$ $\frac{60}{100} = 1-\frac{T_2}{400}$ $\Rightarrow$ $T_2 = \left(1-0.6\right)\times400 = 160K = -113°C$

29.(A) $PT^3 = \text{constant}$

$\Rightarrow$ $KV = T^4$ ...(i)

$k\frac{dV}{dt} = 4T^3$ ...(ii)

Dividing (ii) from (i) $\frac{dV}{Vdt} = \frac{4}{T} = \gamma$

30.(B) $PV = \frac{m}{M}RT$ or $P = \frac{\rho}{M}RT$ $\quad\therefore\quad \frac{P_1}{T_1\rho_1} = \frac{P_2}{T_2\rho_2}$

$\frac{\rho_2}{\rho_1} = \frac{P_2T_1}{P_1T_2} = \frac{45\times300}{76\times266} = \frac{m_2}{m_1}$ (As volume is constant)

$\therefore\quad m_2 = \frac{45\times300\times185}{76\times266} = 123.54kg$

31.(D) $V_{rms} = \sqrt{\frac{3PV}{M}} \propto \frac{1}{\sqrt{M}}$

$\therefore\quad \frac{V_{H_2,rms}}{V_{O_2,rms}} = \sqrt{\frac{32}{2}} = 4$

$V_{H_2,rms} = 4\times V_{O_2,rms} = 4\times160 = 640m\ /\ s$

32.(D) $V_{rms} = \sqrt{\dfrac{3RT}{M}}$; $\quad V_{rms} \propto \dfrac{1}{\sqrt{M}}$

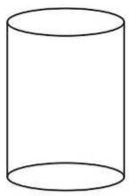
$V_H > V_0 > V_C$

33.(B) $p_{in} = P_{out} + \dfrac{\Delta U}{\Delta t}$

$100 = 90 + \dfrac{2500}{\Delta t}$

$\Delta t = 250\,\text{sec}$

34.(C)



$n_{H_2} = 1$

$n_{CO_2} = 2$

$T = 400\ K$

$PV = nRT$

$P = \dfrac{3 \times 8.3 \times 400}{4 \times 10^{-3}} = 24.9 \times 10^5 Pa$

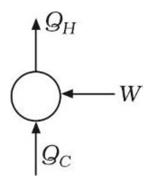
35.(C) $Q_H - Q_C \propto W =$ input energy

$T_H - T_C = 35 \propto W = 35\ J/s$

$263K = T_C \qquad 298K = T_H$

$\dfrac{Q_C}{W} = \dfrac{T_C}{T_H - T_C} = \dfrac{263}{35}$

$\Rightarrow \qquad Q_C = 263\ J/s$



36.(500) $w = 300 - 240 = 60J$

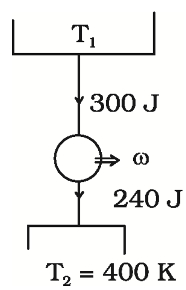
Efficiency (n)

$\eta = \dfrac{\omega}{Q_{input}} = \dfrac{60}{300} = \dfrac{1}{5}$

For minimum temp of hot reservoir engine should be cannot

$\eta_C = 1 - \dfrac{T_2}{T_1}$; $\qquad \dfrac{1}{5} = 1 - \dfrac{400}{T_1}$; $\qquad \dfrac{400}{T_1} = 1 - \dfrac{1}{5}$

$\dfrac{400}{T_1} = \dfrac{4}{5} \Rightarrow T_1 = 500K$



37.(D) $\dfrac{dP}{dV} = -aP$; $\qquad \displaystyle\int_{P_0}^{P} \frac{dP}{P} = -a\int_0^V dV \Rightarrow P = P_0 e^{-aV}$

Now, $PV = RT \quad (\because n = 1 mole)$

$P + V\dfrac{dP}{dV} = \dfrac{RdT}{dV} \qquad (\because \dfrac{dT}{dV} = 0$ as T is maximum)

$P + V\dfrac{dP}{dV} = 0$; $\quad P - aPV = 0$

$V = \dfrac{1}{a}$

Thus $T = \dfrac{PV}{R} = \dfrac{P_0 e^{-1}}{aR} = \dfrac{P_0}{a\,\text{Re}}$

38.(B) $n = 1 - \frac{T_L}{T_M}$

$\frac{1}{4} = 1 - \frac{T_L}{T_M} \Rightarrow \frac{T_L}{T_M} = \frac{3}{4}$ ...(i)

$2n = \frac{1}{2} = 1 - \frac{(T_L - 58)}{T_M} \Rightarrow \frac{T_L - 58}{T_M} = \frac{1}{2}$ ...(ii)

Solving (i) and (ii) we get

$T_L = 174°C$

39.(25) for diatomic gas,

$\Delta Q = n\Delta TC_P = \frac{7Rn\Delta T}{2}$

$\Delta U = n\Delta TC_V = \frac{5nR\Delta T}{2}$

Hence, $\frac{\Delta U}{W} = \frac{\Delta U}{\Delta Q - \Delta U} = 2.5$

x = 25

40.(500) Avg. translational $kE = \frac{3}{2}kT$

k.E of electron = eV

now, $\frac{3}{2}kT = eV$

$T = 773.9$

In °C, $T - 273.15 \simeq 500°C$

41.(A) Number of moles of $O_2 = \frac{m_1}{32}$

Number of moles $H_2 = \frac{m_2}{2}$

Using ideal gas equation $PV = \left[\frac{m_1}{32} + \frac{m_2}{2}\right]\frac{25}{3} \times 300$

$\Rightarrow 200 \times 10^3 \times 1000 \times 10^{-6} = \left[\frac{m_1}{32} + \frac{m_2}{2}\right]25 \times 100$

$\Rightarrow 2 \times 10^2 = \left[\frac{m_1}{m_2} \times \frac{1}{32} + \frac{1}{2}\right]m_2 \times 25 \times 100$

$\therefore \frac{2}{25} = \left[\frac{m_1}{m_2} \times \frac{1}{32} + \frac{1}{2}\right]m_2$ ...(i)

$m_1 + m_2 = 0.76$

$\therefore m_2\left[1 + \frac{m_1}{m_2}\right] = \frac{76}{100}$ ...(ii)

(i) divided by (ii); $\frac{\left[\frac{m_1}{m_2} \times \frac{1}{32} + \frac{1}{2}\right]}{\left[\frac{m_1}{m_2} + 1\right]} = \frac{2}{25} \times \frac{100}{76}$

Let assume $\frac{m_1}{m_2} = x; \frac{x + 16}{32(x+1)} = \frac{8}{76}; 76x + 76 \times 16 = 8 \times 32x + 8 \times 32; 76x + 1216 = 256x + 256$

$960 = 180x \Rightarrow x = \frac{96}{18} \Rightarrow \frac{16}{3}$

42.(480) $P_2 = \left(\frac{1200}{300}\right)^{3/2} 200kPa = 1600kPa$

$$|W_{\text{adiabatic}}| = \frac{P_2V_2 - P_1V_1}{\gamma - 1} = \frac{(1600)(300) - (200)(1200)}{0.5}\left(10^{-3}\right) = 480J$$

43.(C) $\gamma = 1 + \frac{2}{f} \quad \Rightarrow \quad \gamma - 1 = \frac{2}{f} \quad \therefore \quad f = \frac{2}{\gamma - 1}$

44.(C) $\because T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\because \gamma = 1 + \frac{2}{f} = \frac{5}{3} \Rightarrow \gamma - 1 = \frac{2}{3}$

$T_1\left(Al_1\right)^{2/3} = T_2\left(Al_2\right)^{2/3} \Rightarrow \frac{T_1}{T_2} = \left(\frac{l_2}{l_1}\right)^{2/3}$

45.(B) At high temperature and low pressure , a real gas behaves as an ideal gas. For $C_p - C_v = R$ in a state P gas is ideal. So, $T_P > T_Q$

46.(B) $\eta = 1 - \frac{T_C}{T_H} = \frac{1}{6}$ (given)

and $\frac{1}{3} = 1 - \left(\frac{T_C - 62}{T_H}\right) = 1 - \frac{T_C}{T_H} + \frac{62}{T_H}$

$\frac{1}{3} - \frac{1}{6} = \frac{62}{T_H} \quad \Rightarrow \quad T_H = 62 \times 6$

$T_H = 372\ K$ or $T_C = 99°C$

47.(25) $\because$ Mean free path

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$; $\frac{\lambda_1}{\lambda_2} = \frac{d_2^2 n_2}{d_1^2 n_1} = \left(\frac{5}{10}\right)^2 = 0.25 = 25 \times 10^{-2}$

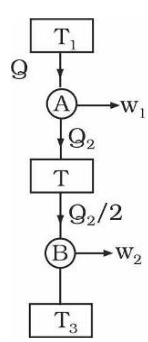
48.(C) Vibrational mode = 4

$\therefore$ Total degree of freedom = 3 + 3 + 2 × 4 = 14

$C_V = 7R$; $C_p = 8R$; $\gamma = \frac{8}{7}$

$W = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{1 \times 8.314(-10)}{1/7} = -582J$

49.(A) $\left(\eta = 1 - \frac{T}{T_1}\right)$



$\omega_1 = Q\left(1 - \frac{T}{T_1}\right)$

$Q_2 = Q - \omega_1 = Q\left(1 - \left(1 - \frac{T}{T_1}\right)\right) = Q\left(\frac{T}{T_1}\right)$

$\eta_2 = 1 - \frac{T_3}{T}$

$\omega_2 = \frac{Q}{2}\left(\frac{T}{T_1}\right)\left(1 - \frac{T_3}{T}\right)$

$\omega_1 = \omega_2$; $2T_1 + T_3 = 3T$

50.(B) $\Delta Q = \Delta U$ (as work done is 0)

$\Delta Q = nC_V \Delta T$

$= 4 \times \left(\frac{5}{2}R\right)(50^\circ - 0^\circ)$

$\Delta Q = 500R$

51.(100) For cyclic process, heat absorbed (Q) = Work done (W)

= [Area under the $P-V$ curve]

$= \frac{\pi\{(40-20)\times 10^3\}\{(40-20)\times 10^{-3}\}}{4} = \frac{\pi \times 20 \times 10^3 \times 20 \times 10^{-3}}{4}$ Joule $= 100\pi$ Joule

52.(B) $PV = nRT$

$\frac{PV}{T} = nR =$ slope $\Rightarrow$ straight line

$PV \propto T$

53.(C) $PV = nRT \quad \Rightarrow \quad 2 \times 1 = n \times 0.0821 \times 300 \quad \Rightarrow \quad n = \frac{2}{24.63}$ Ntotal $\approx \frac{2}{24.63} \times 6.023 \times 10^{23}$.

$\approx 4.9 \times 10^{22}$ molecules

(not there in any option) Gaseous state and thermodynamics

The given kinetic energy per molecule is not correct for the given temperature. NTA has given answer taking that value $2 \times 10^{-9} N = \frac{3}{2}PV = \frac{3}{2}(2 \times 10^5)(10^{-3})$

$N = \frac{3 \times 10^2}{2 \times 10^{-9}} = 1.5 \times 10^{11}$

54.(B) $W_{AB} = \frac{nR\Delta T}{1-v}$

$W_{BC} = \frac{nR\Delta T}{1-v}$

Since, ΔT is same for AD and BC, $\quad \therefore W_{AD} = W_{BC}$

55.(17258) $P_1V_1 = P_2V_2 \Rightarrow PV = nRT$

$W = \int PdV = nRT \int_{V_1}^{V_2} \frac{dV}{V}$

$W = nRT \ln\frac{V_2}{V_1} = (8.3)(300)(\ln 2) J = (2490)(0.6931) J$

$W = 17258 \times 10^{-1} J$

56.(B) $V_{rms} = \sqrt{\frac{3k_BT}{m}}$

$V_{rms} \alpha \frac{1}{\sqrt{m}}$

$m_A < m_B < m_C$

$V_A > V_B > V_C$

$\frac{1}{V_A} < \frac{1}{V_B} < \frac{1}{V_C}$

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1.(C) **Process 1**

P = constant

Volume = increasing

$V \propto T$

Temperature will increase

$\Delta T = +ve$; $\Delta Q = nC_P \, \Delta T = +ve$

Process 2

V = constant pressure = decreasing

$P \propto T$ Temperature = decreasing

$\Delta Q = nC_V \Delta T$ $\Delta T = -ve$ $\Delta Q = -ve$

Process 3

$P = C$ and Vol = decreasing

$P \propto T$ T = decreasing

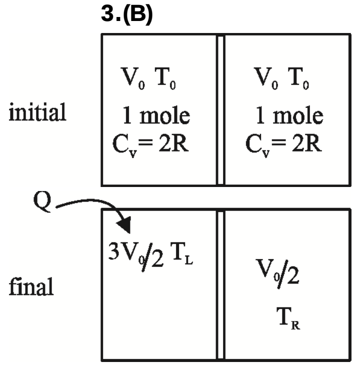
Process 4

$V = C$ P = increasing

$P \propto T$ T = increasing

$\Delta T = +ve$ $\Delta Q = nC_P \, \Delta T = +ve$

2.(A) **3.(B)**



Initial $C_V = 2R$

$C_p = C_v + R = 3R$; $\gamma = \frac{C_p}{C_V} = \frac{3}{2}$

For the right chamber the process is adiabatic ; $T_0 V_0^{1/2} = T_R \left(\frac{V_0}{2}\right)^{1/2} \Rightarrow \frac{T_R}{T_0} = \sqrt{2}$

Work done in right chamber $= -\Delta U = -1 \times 2R \times \left(\sqrt{2}T_0 - T_0\right)$

Work done in left chamber = – Work done in right chamber $= 2\left(\sqrt{2} - 1\right) RT_0$

Final pressure $P_0 V_0^{3/2} = P\left(\frac{V_0}{2}\right)^{3/2} \Rightarrow P = 2\sqrt{2}P_0$

For the left chamber

$\frac{P_0 V_0}{T_0} = \frac{\left(2\sqrt{2}P_0\right)\left(3V_0 / 2\right)}{T_L} \Rightarrow T_L = 3\sqrt{2}T_0$; $\Delta U_L = 1 \times 2R \times \left(3\sqrt{2}T_0 - T_0\right) = 2\left(3\sqrt{2} - 1\right) RT_0$

Using 1st law for left chamber

$Q = \Delta U + \Delta W = 2\left(3\sqrt{2} - 1\right) RT_0 + 2\left(\sqrt{2} - 1\right) RT_0 = 2RT_0\left(4\sqrt{2} - 2\right)$

$Q = 4\left(2\sqrt{2} - 1\right) RT_0$; $\frac{Q}{RT_0} = 4\left(2\sqrt{2} - 1\right)$

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Gaseous State and Thermodynamics	Class - XI | Physics

JEE Main 2022

1.(A) Statement -1 Standard result.

Statement -2 $Q = \Delta U + W_{gas}$ For adiabatic process: $Q = 0$

Since work is done on the gas $W_{gas} \to -ve$

$\therefore\ \Delta U = -\left(w_{gas}\right) = +ve \quad \therefore\ T_f > T_i$

2.(B) $C_P = \left(\frac{f}{2}+1\right)R \qquad C_V\ \frac{fR}{2}$

So $\frac{C_P}{C_V} = \left(1+\frac{2}{f}\right)$

3.(C) $n_{H_2} = \frac{16}{2} = 8$; $n_{O_2} = \frac{128}{32} = \frac{8}{2} = 4$

$PV = nRT$; $\quad V = \frac{nRT}{P}$; $\quad V = \frac{(8+4)(8.314)(273)}{10^5}$

$V = 0.27\ m^3 = 0.27 \times 10^{+6} cm^3$; $\qquad V = 27 \times 10^4\ cm^3$

4.(B) For carnote engine

$\eta = 1 - \frac{T_2}{T_1}$

Case I : $\frac{1}{4} = 1 - \frac{300}{T_1}$

So, $T_1 = 400K$

Case II : New efficiency is increased by 100% means new efficiency become double.

$\eta' = 1 - \frac{T_2}{T_1'}$; $\quad \frac{1}{2} = 1 - \frac{300}{T_1'}$

$\frac{300}{T_1'} = \frac{1}{2}$

$T_1' = 600K$

So, change in temperature of source

$= T_1' - T_1 = 600 - 400 = 200\,K$

or change in temperature increment is 200°C.

5.(12) We know

$v \propto \sqrt{T}$

Case I :

$\Rightarrow\ T = 127 + 273 = 400\,K$

$v \propto \sqrt{400}$ ...(i)

Case II :

For speed to be doubled

$v' = 2v$

$v' \propto 2\sqrt{400}$

$v' \propto \sqrt{1600}$ ...(ii)

$\Rightarrow$ So, $T' = 1600\,K$

$\Rightarrow$ Extra heat required $\Delta Q = \dfrac{fnR\Delta T}{2}$

$\Rightarrow$ For diatomic molecule $f = 5$

$\Rightarrow\ 0.56\,kg = 56\,gm$

= 2 mole of nitrogen

$\Rightarrow\ \Delta Q = \dfrac{5 \times 2 \times 2 \times (1600 - 400)}{2} = 12000 = 12\,Kcal$

6.(C) $\eta = \dfrac{W}{Q_{in}}$

When $\eta =$ efficiency of carnot cycle

$W =$ work done

Q_{in} = heat input

$\Rightarrow\ 1 - \dfrac{T}{T_{source}} = \dfrac{W}{Q_{in}} \Rightarrow\ W = 5000\left(1 - \dfrac{400}{1000}\right)Kcal = 3000\,Kcal$

7.(2) $\omega = \dfrac{Q}{4}$

$Q = \dfrac{Q}{4} + \Delta U \quad \Delta U = \dfrac{3Q}{4}$

$nC_v\Delta T = \dfrac{3Q}{4}; \quad nC\Delta T = Q$

$\dfrac{C_v}{c} = \dfrac{3}{4}; \qquad C = \dfrac{4}{3}C_v = \dfrac{4}{3} \times \dfrac{3}{2}R = 2R$

8.(B) $V_{rms} = \sqrt{\dfrac{3RT}{M}}$

$V_p = \sqrt{\dfrac{2RT}{M}}; \quad v_{rms} = \sqrt{\dfrac{3}{2}}v_p$

9.(250) Since vessel is closed, the process is isochoric.

So, $\dfrac{T_2}{T_1} = \dfrac{P_2}{P_1}$

% age increase in T will be same as

% age increase in P.

Hence, 0.4% of T = 1K

$\dfrac{0.4}{100}.T = 1K$

$\Rightarrow T = 250K$

10.(A) $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{273}{373} = \dfrac{100}{373} \times 100$

11.(B) $\dfrac{1}{2} \times m \times V^2 = \dfrac{5}{2} \times \dfrac{m}{M} \times R \times \Delta T$

$\Delta T = \dfrac{Mv^2}{5R}$

12.(D) Average kinetic energy per molecule $=\frac{f}{2}k_BT$

As argon is monoatomic, its $f = 3$

As oxygen is diatomic, its $f = 5$

$\therefore$ Required ratio $=\frac{3}{5}$

No option matches

13.(540) $\frac{Q_2}{Q_1}=\frac{T_C}{T_H} \Rightarrow \frac{180J}{300J}=\frac{324K}{T_H} \Rightarrow T_H = 540\ K$

14.(B) $pV = nRT$

$$n=\frac{100\left(10^3\right)\left(2000\times10^{-6}\right)}{\left(\frac{25}{3}\right)(300)}=\frac{2}{25}=0.08$$

$n_H(2)+n_0(32)=0.76$; $n_H+16n_0=0.38$

$n_H+n_0=0.08$; $n_H+16n_0=0.38$

$15n_0=0.30$; $n_0=\frac{3}{150}=\frac{1}{50}=0.02$

$n_H+n_0=0.08$; $n_H+0.02=0.08$

$n_H=0.06$; $n_H:n_O=3:1$

15.(16) $T_1=527°C \Rightarrow T_1=800K$

$T_2=200K$

$$\eta=\frac{T_1-T_2}{T_1}=\frac{800-200}{800}=\frac{3}{4}$$

$$w=Q_{ab}\times\eta\Rightarrow Q_{ab}=\frac{12000kJ}{3/4}$$

$$Q_{ab}=16\times10^6 J$$

16.(A) $PV=nRT$; $V=\left(\frac{nR}{P}\right)T$

Slope = $\frac{nR}{P}$

Slope $\propto \frac{1}{P}$ $\therefore$ $P_1>P_2$

17.(B) Average Kinetic energy per molecule / degree of freedom $=\frac{k_BT}{2}$

Gas molecules freeze at 0 K (not at 0°C)

Remaining options are correct.

18.(1400) Process is isobaric

$W=nR\Delta T=400J$

$$\therefore \quad \Delta Q=nC_r\Delta T=n\frac{\gamma R}{\gamma-1}\Delta T=\frac{400\times1.4}{0.4}=1400J$$

19.(7479) $U=nC_vT=2\times\frac{R}{\left(\frac{5}{3}-1\right)}\times300=3\times8.31\times300=7479J$

20.(B) Total heat = Work done by gas + ΔU

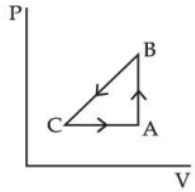
$\Delta U = 0$ for cyclic process

$40 + (-60) = W_{CA} + W_{AB} + W_{BC}$

$\Rightarrow \quad -20 = W_{CA} + 0 + (-50)$

$\Rightarrow \quad -20 + 50 = W_{CA}$

$\Rightarrow \quad 30J = W_{CA}$



21.(B) $(V_{rms})_{O_2} = \sqrt{\dfrac{3RT}{M_0}}$

When it was O_2 molecule $(V_{rms})_0 = \sqrt{\dfrac{3R(2T)}{M_0/2}}$

$(V_{rms})_0 = \sqrt{\dfrac{3R(2T)}{M_0/2}}$; $\quad (V_{rms})_0 = 2(V_{rms})_{O_2}$

22.(C) n = no. of moles = 2

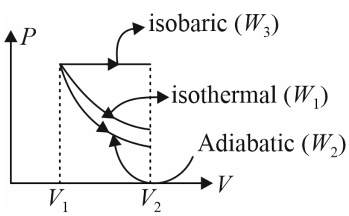
V = const

$\Delta Q = nC_V\Delta T = 2\times 3\times \dfrac{R}{2}\times \Delta T$

$= 2\times 3\times \dfrac{8.3}{2}\times 20 = 498J$

23.(102) $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2} \Rightarrow T_2 = \dfrac{500\times 225}{300} = 375K \quad = 102°C$

24.(D)



Area under P-V graph is work done by gas

$\therefore \quad W_3 > W_1 > W_2$

25.(B) $P_1 = 2\times 10^7\ N/m^2$

$P_1V_1 = P_2V_2$

Given : $V_2 = 2V_1$

$P_1V_1 = P_2(2V_1)$

$\Rightarrow \quad P_2 = \dfrac{P_1}{2} = \dfrac{2\times 10^7}{2} = 10^7\ N/m^2$

After isothermal process, the gas expands adiabatically

$\therefore \quad P_2V_2^{\gamma} = P_3V_3^{\gamma} \quad \Rightarrow \quad P_2V_2^{\gamma} = P_3(2V_2)^{\gamma}$

$\Rightarrow \quad P_3 = \dfrac{P_2}{2^{\gamma}} = \dfrac{10^7}{2^{1.5}} = 3.53\times 10^6 N/m^2$

26.(A) Only statement I is correct and Option A says statements I and IV are correct.

27.(B) $n_1 = 1 - \dfrac{(147+273)}{447+273} = \dfrac{300}{720} = \dfrac{5}{12}$

$n_2 = 1 - \dfrac{320}{1220} = \dfrac{900}{1220} = \dfrac{45}{61}$

$$\frac{n_1}{n_2}=\frac{5}{12}\times\frac{61}{45}=\frac{61}{108}\approx 0.56$$

28.(B) $\Delta U=\frac{fnR\Delta T}{2}$; $\Delta U=\frac{7\times 3\times 8.3\times 40}{2}$

$\Delta U=3486$ Joule

29.(C) $PV^{\gamma}=$ constant

$\gamma=1+\frac{2}{f}$; $\gamma=1+\frac{2}{3}=\frac{5}{3}$

$PV^{\gamma}=P'\left(\frac{V}{8}\right)^{\gamma}$; $PV^{5/3}=P'\left(\frac{V}{8}\right)^{5/3}$

$PV^{5/3}=P'\frac{V^{5/3}}{8^{5/3}}$; $P=\frac{P'}{32}$; $\boxed{P'=32P}$

30.(A) $\frac{C_P}{C_V}=1+\frac{2}{n}\Rightarrow\frac{C_V}{C_P}=\frac{n}{(n+2)}$

31.(A) For option B

$Q=\sigma AT^4$

$\frac{(283)^4}{(293)^4}=\frac{1}{1.15}$

(A) and (C) are theory based

32.(C) $V_{rms}=\sqrt{\frac{3RT}{M}}$

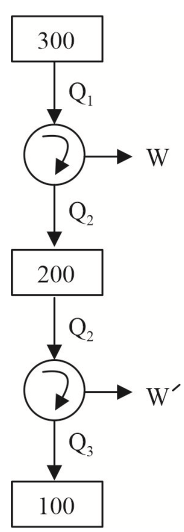
$\because$ Temperature and Molar mass are same

$\therefore$ V_{rms} of both sample will be same $\boxed{A}$

$PV=nRT$; $P\propto n$

$\therefore$ $\frac{P_1}{P_2}=\frac{n_1}{n_2}=1:4$ $\boxed{B}$

33.(A) As two Carnot engines are connected in series in case 2, so the heat rejected by first Carnot engine is always taken as heat injected in second Carnot engine:



In case 2:

$\because$ $\frac{Q_2}{Q_1}=\frac{200}{300}\Rightarrow Q_2=\frac{2Q_1}{3}$

$\because$ $\frac{Q_3}{Q_2}=\frac{100}{200}\Rightarrow Q_3=\frac{Q_2}{2}$

$\Rightarrow$ $\eta_{eff}=1-\frac{Q_3}{Q_1}=1-\frac{1}{3}=\frac{2}{3}$

In Case 1:

$\Rightarrow$ $\eta=1-\frac{100}{300}=\frac{2}{3}$

34.(B) As per maxwell law, each degree of freedom has $\frac{1}{2}KT$ average translational energy associated with it.

CH_4 has 3 translational and 3 rotational degree of freedom.

35.(C) $\eta = \left(1 - \dfrac{T_2}{T_1}\right) \times 100$

(i) When $\eta = 50\% \Rightarrow 0.5 = \left(1 - \dfrac{T_2}{T_1}\right) \Rightarrow \quad T_2 = 0.5T_1$

(ii) When η is increased by 30%. It becomes $\left(50 + \dfrac{50 \times 30}{100}\right)\% = 65\%$

So $\Rightarrow \quad 0.65 = \left(1 - \dfrac{T_2 - 40}{T_1}\right) \Rightarrow T_2 - 40 = 0.35T_1$

$\Rightarrow \quad 0.5T_1 - 0.35T_1 = 40 \quad \Rightarrow \quad 0.15T_1 = 40 \Rightarrow T_1 = \dfrac{4000}{15}; \quad T_1 = 266.7K$

36.(D) (I) Average momentum of all particle = zero always at any temperature.

$\because$ Particles moves in random direction at all temperatures.

(II) $V_{rms} = \sqrt{\dfrac{3RT}{M}}$

So, $V = \sqrt{\dfrac{3RT_0}{M_0}}$ and $V_2 = \sqrt{\dfrac{3R(2T_0)}{\left(\dfrac{M_0}{2}\right)}} = 2V$

37.(C) $\theta = nc_V \Delta T$ ... (i)

$v_{rms} = \sqrt{\dfrac{3RT}{M}}$ Hence to double rms vel

$T_i = 300K, \ T_f = 1200K$

Now $\theta = \dfrac{1}{2} \times \dfrac{5}{2} \times 8.3 \times 900 = 9360$

38.(750) Given

$W = P(V_2 - V_1) = 150 = nR\Delta T$ ... (i)

$Q = nC_P \Delta T = \dfrac{n(f+2)R}{2} \Delta T = 5nR\Delta T$ ... (ii)

$\Rightarrow \quad Q = 150 \times 5 = 750\, J$

39.(4) For adiabatic process,

$T V^{\gamma - 1} = \text{constant}$

$T \left(\dfrac{m}{d}\right)^{\gamma-1} = \text{constant}$

$T d^{1-\gamma} = \text{constant} \quad \Rightarrow \quad T_1 d_1^{1-\gamma} = T_2 d_2^{1-\gamma} \quad \therefore \quad T_2 = 4T_1$

40.(3) At constant volume,

$n_1 (C_v)_1 \Delta T + n_2 (C_v)_2 \Delta T = (n_1 + n_2)(C_v)_{mix} \Delta T$

$\therefore \quad (C_v)_{mix} = \dfrac{n_1 (C_v)_1 + n_2 (C_v)_2}{n_1 + n_2} = \dfrac{\left(1 \times \dfrac{3R}{2}\right) + \left(3 \times \dfrac{5R}{2}\right)}{1 + 3}$

or, $\dfrac{\alpha^2}{4} R = \dfrac{18R}{8} \quad \Rightarrow \quad \alpha = 3$

41.(B) Area under graph $\rightarrow \dfrac{1}{2}(3)(300) = 450(J)$

42.(C) $V_{r.m.s} = \sqrt{\dfrac{3KT}{m}} = 15 mm\ / sec$

Solutions of Archive - JEE Main & Advanced

Simple Harmonic Motion	Class - XI | Physics

JEE Main 2021

1.(B) New angular frequency $=\omega_n=\sqrt{\dfrac{k}{M+m}}$

If new Amplitude $=A_n$, then using conservation of linear momentum:

$M(A\omega)=(M+m)A_n\omega_n$

$$\Rightarrow \qquad A_n=A\frac{\omega}{\omega_n}\times\frac{M}{M+m}$$

$$=A\times\sqrt{\frac{k}{M}}\times\sqrt{\frac{M+m}{k}}\times\frac{M}{M+m} \qquad =A\sqrt{\frac{M}{M+m}}$$

2.(D) The frequency of the SHM depends only on the mass and the springs, not the orientation of the arrangement.

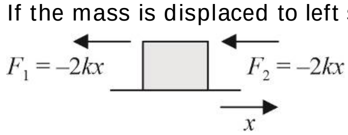
Therefore, the frequency, $f=\dfrac{1}{2\pi}\left(\dfrac{2k}{m}\right)^{1/2}$

3.(C) $v^2=w^2(A^2-x^2) \qquad \Rightarrow \qquad \dfrac{v^2}{A^2w^2}+\dfrac{x^2}{A^2}=1$

An ellipse.

4.(A) $T=2\pi\sqrt{\dfrac{l}{g}}\Rightarrow 2\pi\sqrt{\dfrac{2}{g}}=2 \qquad \therefore \quad g=2\pi^2\,ms^{-2}$

5.(D) If the mass is displaced to left side by a small direction x.



$\vec{F}=-4k\vec{x}$; $\qquad \vec{a}=-\dfrac{4k}{m}\vec{x}$

$\therefore \qquad T=2\pi\sqrt{\dfrac{m}{4k}} \qquad$ or $\qquad T=\pi\sqrt{\dfrac{m}{k}}$

6.(B) $y=A\sin(\omega t+\omega_0)$; $\qquad v=\dfrac{dy}{dt}=A\omega\cos(\omega t+\omega_0)$

at $t=0$, $y=\dfrac{A}{2}$; $\qquad \sin\omega_0=\dfrac{1}{2}$; $\qquad \omega_0=\dfrac{\lambda}{6}$ or $\dfrac{5\lambda}{6}$

But v is along negative y direction at $t=0$.

$\therefore \qquad \cos\omega_0$ should be negative ; $\qquad \omega_0=\dfrac{5\lambda}{6}$.

***Note :** In the question it is mentioned that the particle is moving along x-axis but time-displacement is given for y-axis.

7.(C) $\left|\vec{F_r}\right| = m\omega^2 R$; $\dfrac{\left|\vec{F_r}\right|}{m} = \omega^2 R$; $\omega = \dfrac{2\pi}{T}$

where T is time period. $T = 0.1 \times 12 = 1.2$ seconds

$\therefore \quad \dfrac{\left|\vec{F_R}\right|}{m} = \left(\dfrac{2\pi}{1.2}\right)^2 \times 0.36 \quad = 9.87$ N

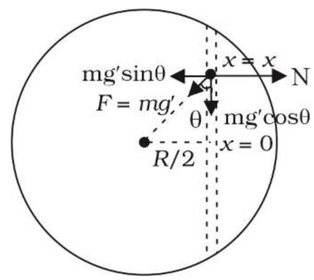
8.(A) $\dfrac{1}{K_{eqv}} = \dfrac{1}{K_1} + \dfrac{1}{K_2} = \dfrac{1}{K_1} + \dfrac{1}{K_1}$ $\quad [K_1 = K_2]$

$K_{eqv} = \dfrac{K_1}{2}$

$\therefore \quad T_{new} = 2\pi\sqrt{\dfrac{m}{K_{eqv}}} = 2\pi\sqrt{\dfrac{m}{\frac{K_1}{2}}} = \sqrt{2} \times \left(2\pi\sqrt{\dfrac{m}{K_1}}\right)$

9.(A) At any general position $x = x$ w.r.t mean position $x = 0$

$F_{restoring} = mg' \cos\theta = m \times g\left(1 - \dfrac{h}{R}\right) \times \dfrac{x}{(R-h)}$



Where $R - h = \sqrt{\left(\dfrac{R}{2}\right)^2 + x^2}$

or $\quad F_{restoring} = \left(\dfrac{mg}{R}\right)x = Kx$

$\therefore \quad T = 2\pi\sqrt{\dfrac{m}{K}} = 2\pi\sqrt{\dfrac{m}{mg/R}} = 2\pi\sqrt{\dfrac{R}{g}}$

10.(C) Velocity position graph for particle in SHM.

$\left[v = \omega\sqrt{A^2 - x^2}\right]$**;** $\quad \dfrac{V^2}{A^2\omega^2} + \dfrac{x^2}{A^2} = 1$

11.(B) Fact

12.(3) $x = A\sin\omega t$ and $v = \underbrace{A\omega}_{v_{\max}} \cos\omega t \quad \Rightarrow \quad v = \omega\sqrt{A^2 - x^2}$

$v = \dfrac{A\omega}{2} = \omega\sqrt{A^2 - x^2} \quad \Rightarrow \quad x = \pm\dfrac{\sqrt{3}a}{2}$ $\quad$ Thus, $x = 3$

13.(7) In terms of angular displacement particle will be at $\dfrac{\theta_{\max}}{2}$ after completing half oscillation since

$\dfrac{5}{8} = \dfrac{1}{\underbrace{2}_{\text{half}}} + \dfrac{1}{\underbrace{8}_{\text{1 eight}}}$

For half oscillation $\Delta t_1 = T/2$ and from mean position to $\dfrac{\theta_{\max}}{2}$ position $\Delta t_2 = \dfrac{\pi/6 - 0}{\omega} = \dfrac{T}{12}$

Thus total time taken $= \Delta t_1 + \Delta t_2 = \dfrac{7T}{12} = \dfrac{\alpha}{\beta}T \quad \Rightarrow \quad \alpha = 7$

14. **No option matching**

Amplitude, $\quad A = A_0 e^{-bt/2m}$

$6 = 12\, e^{-60b}$

$\Rightarrow \quad 60b = \ln 2 \quad \Rightarrow \quad b = \dfrac{0.693}{60} = 1.16 \times 10^{-2}\ kg\ s^{-1}$

15.(B) $A_1\omega_1 = A_2\omega_2$

$A_1\sqrt{K_1} = A_2\sqrt{K_2}$

$\frac{A_1}{A_2} = \sqrt{\frac{K_2}{K_1}}$

16.(B) $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow \quad m\omega^2\left(A^2 - x^2\right) = kx^2 \left[\because v = \pm\omega\sqrt{A^2 - x^2}\right] \qquad \because \quad k = m\omega^2$

$\Rightarrow \quad A^2 - x^2 = x^2 \quad \Rightarrow \quad 2x^2 = A^2 \quad \Rightarrow \quad x = \pm\frac{A}{\sqrt{2}}$

17.(2) $T_a = 2\pi\sqrt{\frac{M}{k}}$

$T_b = 2\pi\sqrt{\frac{M}{k_{eq}}} \qquad \therefore \qquad \frac{T_b}{T_a} = \sqrt{\frac{k}{k_{eq}}} = \sqrt{\frac{k}{k/2}} = \sqrt{2}$

18.(B) $T = 2\pi\sqrt{\frac{l}{g}}$; $\quad T' = 2\pi\sqrt{\frac{l}{g + \frac{g}{2}}} = \sqrt{\frac{2}{3}}T$

19.(6) $\Delta t = \frac{\Delta\phi}{\omega}$ and $\Delta\phi = \frac{\pi}{6},\ \omega = \frac{2\pi}{T}$**;** $\qquad \Delta t = \frac{T}{12} = \frac{2}{12} = \frac{1}{6}\text{sec}$

20.(1 or 4) Angular frequency 2ω

21.(C) $A = A_0 e^{\frac{-bt}{2m}}$; $b = 20g/s,\ m = 500g$

$\frac{A_0}{2} = A_0 e^{\frac{-bt}{2m}} \quad \Rightarrow \quad e^{\frac{bt}{2m}} = 2$

$\Rightarrow \quad \frac{bt}{2m} = \ln 2 = 0.693 \quad \Rightarrow \quad t = \frac{0.693 \times 2m}{b} = \frac{0.693 \times 2 \times 500}{20} \approx 34.65\,\text{sec}$

22.(2) $x_2 = 5 \times 2\left[\frac{1}{\sqrt{2}}\sin(2\pi t) + \frac{1}{\sqrt{2}}\cos(2\pi t)\right] = 10\sin\left(2\pi t + \frac{\pi}{4}\right)$; $\quad \frac{A_2}{A_1} = \frac{10}{5} = 2$

23.(B) $x = x_0 \sin\omega t$

$P.E. = \frac{1}{2}kx^2 \qquad = \frac{1}{2}kx_0^2\sin^2\omega t$

$P.E. = \frac{1}{2}kx_0^2\sin^2\omega t$

24.(1) $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

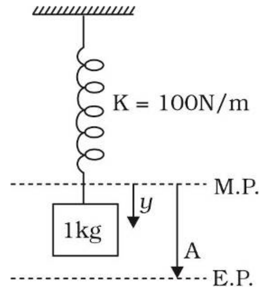
$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right) = 5\left(\frac{2}{2}\sin 3\pi t + \frac{2\sqrt{3}}{2}\cos 3\pi t\right)$

$= 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right) = 10\left(\cos\frac{\pi}{3}\sin 3\pi t + \sin\frac{\pi}{3}\cos 3\pi t\right) = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

So ratio of amplitude of y_1 to y_2 is $10 : 10 \Rightarrow 1 : 1$

So the value of $x = 1$

25.(8) $T = 2\pi\sqrt{\frac{m}{K}}$



$(P.E.)_{max} = \frac{1}{2}KA^2$

When,

$\Rightarrow \quad K.E. = P.E. \quad \Rightarrow \quad P.E. = \frac{(P.E.)_{max}}{2}$

This happens at

$\therefore \quad y = \frac{A}{\sqrt{2}} \qquad \therefore \quad t = \frac{T}{8} = \frac{T}{x}$

$x = 8$

26.(D) $U_{max} = 10$

$\frac{1}{2}m\omega^2 A^2 = 10$, here $A = 2$

$\omega = 1$, also $\omega = \sqrt{\frac{g}{l}}$

$g = \frac{4m}{S^2}$

27.(D) $g_{effective} = \frac{mg - F_B}{m} = \frac{\rho vg - \frac{1}{4}\rho vg}{\rho v}$

$g_{effective} = \frac{3}{4}g$

$l_{New} = l + \frac{l}{3} = \frac{4l}{3}$; $\qquad T = 2\pi\sqrt{\frac{l}{g}}$

$T_{New} = 2\pi\sqrt{\frac{\frac{4l}{3}}{\frac{3}{4}g}} = 2\pi\sqrt{\frac{16l}{9g}} = \frac{4}{3} \times 2\pi\sqrt{\frac{l}{g}}$; $\qquad T_{New} = \frac{4}{3}T$

28.(C) **(c)** $KE + PE = \text{constant}$

(d) $\underset{0-T}{\langle KE \rangle} = \underset{0-T}{\langle P.E. \rangle} = \frac{mA^2w^2}{4}$

29.(A) $TE = \frac{1}{2}KA^2 = PE + KE$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$

$KE = \frac{1}{2}K\frac{3}{4}A^2 = \frac{3}{4}TE \qquad (m\omega^2 = K)$

30.(C) $(KE)_y = \frac{3}{4}E$

But $E = KE_{max} = \frac{1}{2}m\omega^2 a^2$

$(KE)_y = \frac{3}{4} \times \frac{1}{2}m\omega^2 a^2 = \frac{3}{8}m\omega^2 a^2$

But $(KE)_y = \frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}m\omega^2(a^2 - y^2) = \frac{3}{8}m\omega^2 a^2$

$a^2 - y^2 = \frac{3}{4}a^2$; $\qquad y^2 = \frac{a^2}{4} \quad \Rightarrow \quad y = \frac{a}{2}$

31.(C) $m = \frac{1}{2} kg$; $A = \frac{5}{100} m$

$T = 0.2 \sec$

$\omega = \frac{2\pi}{T} = 10\pi \quad rad/s$

At time $t = \frac{T}{4}$

PE = max = K.E Max

$= \frac{1}{2} m \,|A\omega|^2 \quad = .62$ J

32.(2) $v = w\sqrt{A^2 - x^2}$

$2w = w\sqrt{A^2 - 2^2}$

$A^2 - 4 = 4$

$A^2 = 8$

$A = 2\sqrt{2}$ cm

33.(C) $l \to l/16$; $T \alpha \sqrt{l}$

$T_0 \to T_0/4$

34.(A) $x = C\cos(\omega t - \phi)$

at $t = 0$, $x_0 = C \cos\phi$ ... (i)

$v = -c\omega \sin(\omega t - \phi)$

At $t = 0$, $v_0 = C\omega \sin\phi$ ... (ii)

from (i) & (ii)

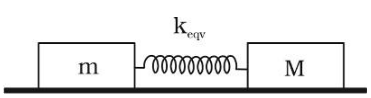
$C = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$; $\tan\phi = \frac{v_0}{x_0 \omega}$

35.(D) $v = \pm\omega\sqrt{A^2 - x^2}$

$A = \sqrt{\frac{v^2}{\omega^2} + x^2} \quad \therefore \quad \sqrt{\frac{v_1^2}{\omega^2} + x_1^2} = \sqrt{\frac{v_2^2}{\omega^2} + x_2^2}$

$\Rightarrow \quad \frac{v_1^2}{\omega^2} + x_1^2 = \frac{v_2^2}{\omega^2} + x_2^2 \quad \Rightarrow \quad v_1^2 - v_2^2 = \left(x_2^2 - x_1^2\right)\omega^2$

$\Rightarrow \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \quad \therefore T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

36.(10) 

$k_{eqv.} = \frac{k_1 k_2}{k_1 + k_2} = \frac{k \times 4k}{k + 4k} = \frac{4}{5} k$

$\therefore \omega = \sqrt{\frac{k_{eqv.}}{m_{reduced}}} = \sqrt{\frac{\frac{4}{5}k}{\left(\frac{m_1 m_2}{m_1 + m_2}\right)}} = \sqrt{\frac{4}{5} \times \frac{20}{0.16}} = 10$ rad/s $\left[m_{reduced} = \frac{m_1 m_2}{m_1 + m_2} = \frac{0.2 \times 0.8}{0.2 + 0.8} = 0.16\right]$

37.(D) $p = F.v = mav$

$\Rightarrow \quad av = \frac{p}{m} \quad \Rightarrow \quad vdv = \frac{p}{m}dt$

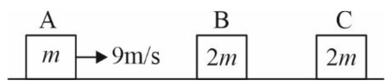
$\Rightarrow \quad \int vdv = \frac{p}{m}\int dt \quad \Rightarrow \quad \frac{v^2}{2} = \frac{p}{m}t$

$\Rightarrow \quad v = \sqrt{\frac{2p}{m}}\sqrt{t} \quad \Rightarrow \quad \int ds = \sqrt{\frac{2p}{m}}\int \sqrt{t}.dt \quad \therefore \quad s = \frac{2}{3}\sqrt{\frac{2p}{m}}\,t^{3/2}$

38.(C) $9m = mV_A + 2mV_B$ ... (i)

$9 = V_A + 2V_B$ ... (i)

$9 = V_B - V_A$ ... (ii)



$18 = 3V_B$

$V_B = 6m\ /\ s$

Collision between B and C is completely inelastic

$\frac{V_B}{2} = V_C \quad \Rightarrow \quad V_C = 3m\ /\ s$

39.(C) In case (a): change in momentum $= mu - (-mu) = 2mu$

In case (b); change in momentum $= mu\cos 45° - (-mu\cos 45°) = \sqrt{2}mu$

$\Rightarrow Ratio = \sqrt{2} : 1$

40.(1) $K = 100N\ /\ m \quad m = 100g \quad \Delta l = 0.05m$

$h = 2m$

Let velocity of ball when it leaves the barrel of gun be 'u'

By conservation of energy for ball

$\frac{1}{2}K\ \Delta l^2 = \frac{1}{2}mu^2$; $\quad u = \Delta l\sqrt{\frac{K}{m}} = 0.05\sqrt{\frac{100}{400 \times 10^{-3}}}$

$u = 0.5\sqrt{10} = \frac{\sqrt{10}}{2}m\ /\ \sec$

For horizontal projection from height to ground,

$d = u\sqrt{\frac{2h}{g}} = \frac{\sqrt{10}}{2}\sqrt{\frac{2 \times 2}{10}}$

$d = 1m$ so Answer = 1

41.(C) Conserving momentum

$P_{gun} = P_{bullet}$

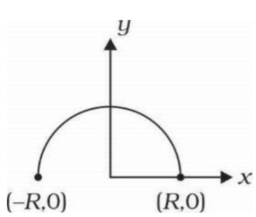
$= 0.004 \times 50 = 0.2\ kg\ m\ /\ s$

$V_{gun} = \frac{0.2}{4} = 0.05m\ /\ s$

42.(C) Word done $= mg \times h \times \cos 180° \quad = 80 \times 9.8 \times 0.8 \times (-1) = -64 \times 9.8 = -627.2J$

43.(2) $y_{cm} = \frac{2R}{\pi}$

$|x| = 2$



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Solutions of Archive - JEE Main & Advanced

Simple Harmonic Motion	Class - XI | Physics

JEE Main 2022

1.(B) As we know, $V_{mass} = A\omega$

$\Rightarrow A_1\omega_1 = A_2\omega_2$ (since both have equally maximum velocities)

$\Rightarrow \dfrac{A_1}{A_2} = \dfrac{\omega_2}{\omega_1} = \sqrt{\dfrac{k_2}{m_2} \times \dfrac{m_1}{k_1}}$

$= \sqrt{\dfrac{9}{2} \times \dfrac{1}{2}} = \dfrac{3}{2} \quad \Rightarrow A_1 : A_2 = 3 : 2$

2.(C) $T = 2\pi\sqrt{\dfrac{L}{g}}$

$T' = 2\pi\sqrt{\dfrac{L}{g + g/6}} = 2\pi\sqrt{\dfrac{6L}{7g}} = \sqrt{\dfrac{6}{7}}T$

3.(D) $x = A\sin(\omega t)$; $\quad \dfrac{A}{2} = A\sin(\omega t)$

$\sin(\omega t) = \dfrac{1}{2}$

$\omega t = \dfrac{\pi}{6} \Rightarrow \dfrac{2\pi}{T}t = \dfrac{\pi}{6}$

$T = 12t = 12(3) = 36\,\text{sec}$

4.(B) $x = \sin\left(\pi t + \dfrac{\pi}{3}\right)$

$\dfrac{dx}{dt} = v = \pi\cos\left(\pi t + \dfrac{\pi}{3}\right)$

$v(at\ t = 1s) = -\dfrac{\pi}{2} m/s$; $\quad |\vec{v}| = 1.57\, m/s$ or $157\, cm/s$

5.(1) Time taken from A to $\dfrac{A}{2} = \dfrac{T}{6} = \dfrac{6}{6} = 1\,\text{sec}$

6.(700) $v = w\sqrt{A^2 - x^2}$

$v' = w\sqrt{A'^2 - x^2} = 3v$ (After velocity is tripled)

$\Rightarrow \quad 9 = \dfrac{A'^2 - x^2}{A^2 - x^2}$

$A'^2 - 25 = 9(100 - 25)$; $\quad A'^2 = 900 - 225 + 25 = 700$

$A' = \sqrt{700}$

7.(C) $\omega = \pi$; $\dfrac{2\pi}{T} = \pi$

$T = 2$; $2\pi\sqrt{\dfrac{\ell}{g}} = 2$

$\pi^2 \dfrac{\ell}{g} = 1$; $\ell = \dfrac{g}{\pi^2}$; $\ell = 99.4\ cm$

8.(A) $T_1 = 2\pi\sqrt{\dfrac{3m}{2k}}$; $T_2 = 2\pi\sqrt{\dfrac{m}{3k}}$ $\therefore$ $\dfrac{T_1}{T_2} = \dfrac{3}{\sqrt{2}}$

9.(D) At a height $h = 2R$ from earth surface

$g_{eff} = \dfrac{GM}{(3R)^2} = \dfrac{GM}{9R^2} = \dfrac{g}{9}$

$T = 2\pi\sqrt{\dfrac{l}{g}}$ $= 2\pi\sqrt{\dfrac{l \times 9}{g}}$

$2 = 2\pi\sqrt{\dfrac{9l}{\pi^2}}$; $l = \dfrac{1}{9}m$

10.(B) $v = \omega\sqrt{a^2 - x^2}$; $v^2 = \omega^2\left(a^2 - x^2\right)$

$\dfrac{v^2}{\omega^2} = a^2 - x^2$; $\dfrac{v^2}{\omega^2} + x^2 = a^2$

$\dfrac{v^2}{\omega^2 a^2} + \dfrac{x^2}{a^2} = 1$ Ellipse

11.(2) In figure (a)

Spring are in series combination

$k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2} = \dfrac{2k}{3}$

$T_1 = 2\pi\sqrt{\dfrac{3m}{2k}} = 3$ secs (given ...(1)

in figure (b)

springs are in parallel combination

$k_{eq} = k_1 + k_2 = 3k$

$T_2 = 2n\sqrt{\dfrac{m}{3k}} = \sqrt{x}$ ...(2)

Equating (1) and (2) $x = 2$

12.(16) $m_1 = 900\ gm$

$m_f = m_1 + m_2 = 900 + 124 = 1024\ gm$

Momentum by body $= mv$

$= m\omega A$

$= m\sqrt{\dfrac{k}{m}}A = \sqrt{mk}\,A$

Conservation of momentum

$$\frac{A_1}{A_2} = \frac{\sqrt{m_1 + m_2}}{\sqrt{m_1}}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{1024}{900}} = \frac{32}{30} = \frac{16}{15} = \frac{16}{16-1}; \qquad \alpha = 16$$

13.(C) Given, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T_1 = 2\pi\sqrt{\frac{\ell}{g'}}$; $\frac{T}{T_1} = \sqrt{\frac{g'}{g}} = \frac{4}{6} = \frac{2}{3}$

Hence $\frac{g'}{g} = \frac{4}{9}$

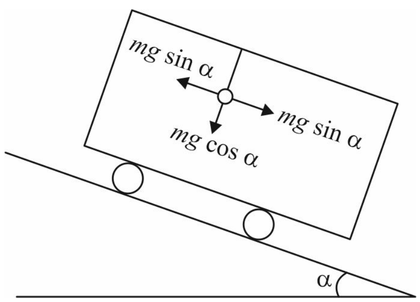
$$\frac{\frac{GM}{(R+x)^2}}{\frac{Gm}{R^2}} = \frac{4}{9} \Rightarrow \frac{R^2}{(R+x)^2} = \frac{4}{9}; \quad x = \frac{R}{2} = 3200\,Km$$

14.(2) $F = \frac{-dU}{dx} = -4\times 4\sin 4x = -16\sin 4x \simeq -64x$

Now $-m\omega^2 x = -64x$; $m\omega^2 = 64$

$\omega = \sqrt{\frac{64}{m}}$, $T = 2\pi\sqrt{\frac{m}{64}} = \frac{\pi}{K}$; Hence $k = 2$

15.(A)



$g_{eff} = g\cos\alpha$

$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{g\cos\alpha}}$

16.(5) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$ $g \to$ apparent gravitation acceleration

$g_1 = g$ **;** $g_2 = g - \frac{g}{5} = \frac{4g}{5m}$

$\frac{10}{T} = \sqrt{\frac{4\frac{g}{5}}{g}} = \sqrt{\frac{4}{5}}$ **;** $T = 10\sqrt{\frac{5}{4}} = 5\sqrt{5}$ sec

$x = 5$ sec

Solutions of Archive - JEE Main & Advanced

Wave Motion	Class - XI | Physics

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1.(A) To represent a travelling wave, a function must be reducible to a function of $(x - vt)$

Here, x is the distance along the direction of propagation, t is time and v is the wave speed (a constant)

2.(8) Frequency of sound heard due to own horn $= f_0$

Frequency of sound heard due to the other horn, $f = f_0\left(\dfrac{c-(-v)}{c-v}\right) = f_0\left(\dfrac{c+v}{c-v}\right)$

So, beat frequency, $f_B = f - f_0 = f_0\left(\dfrac{c+v}{c-v} - 1\right)$

$= f_0\left(\dfrac{2v}{c-v}\right) \qquad = 676\left(\dfrac{2\times 2}{340-2}\right) = 8$ Hz

3.(1215) $k = 1, \ \omega = 30$

$k = \dfrac{\omega}{V} \quad \Rightarrow \quad V = \dfrac{\omega}{k} = 30\,m/s$

$\mu = 0.135\,g/cm = \dfrac{0.135\times 10^{-3}kg}{10^{-2}m} = 1.35\times 10^{-2}kg/m$

$V = \sqrt{\dfrac{T}{\mu}} \Rightarrow T = \mu V^2 = 1.35\times 10^{-2}\times 900\ N = 1215\times 10^{-2}N \qquad \therefore \quad x = 1215$

4.(C) $340 - f = 5$

$f = 335\,Hz$

5.(2) $v = \sqrt{\dfrac{T}{M}} \Rightarrow \dfrac{dv}{v}\times 100 = \dfrac{1}{2}\dfrac{dT}{T}\times 100 = \dfrac{1}{2}\times 4 = 2\%$

6.(D) $A = \dfrac{0.06}{2} = 0.03\,m$

$\omega = 2\pi f = 2\times 3.14\times 245 = 1.5\times 10^3$

$K = \dfrac{\omega}{v} = \dfrac{1.5\times 10^3}{300} = 5.1$

7.(4) $V_1 = \sqrt{\dfrac{B}{\rho_1}};\ V_2 = \sqrt{\dfrac{B}{\rho_2}} \qquad \Rightarrow \qquad \dfrac{V_1}{V_2} = \sqrt{\dfrac{\rho_2}{\rho_1}} \qquad \ldots$ (i)

Same frequency pipes for both $= f = \dfrac{3V_1}{4L} = \dfrac{2V_2}{2L'}$

$\Rightarrow \qquad L' = \dfrac{4LV_2}{3V_1} = \dfrac{4}{3}L\sqrt{\dfrac{\rho_1}{\rho_2}} \qquad \Rightarrow \qquad x = 4$

8.(2025)

$$f = f_0\left(\frac{V - V_0}{V + V_5}\right)$$

$$180 = f_0\left(\frac{340-20}{340+20}\right)$$

$$f_0 = 2025Hz$$

9.(1) $K = 1.57$

$2\pi\lambda = 1.57$

$\lambda = 4cm$

$$\frac{\lambda}{4} = 1cm$$

10.(1210) $V_x = 36km / hr = 10m / \sec$

$V_y = 72km / hr = 20m / \sec$

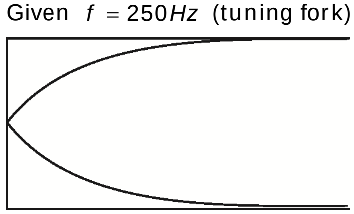
$\xrightarrow{10}$ $\quad$ $\xleftarrow[20]{}$

x $\quad$ y

$$f_y = \left(1320 = \left(\frac{340+20}{340-10}\right)\times f_0\right)$$

$$\frac{1320}{36}\times 33 = f_0 \Rightarrow 1210\,Hz$$

11.(34) Given $f = 250Hz$ (tuning fork)



$f = \dfrac{(2n-1)v}{4L}$ (Frequency of close organ pipe)

For shortest length

$n = 1 \Rightarrow f \Rightarrow$ fundamental frequency

$$f = \frac{v}{4L} = 250$$

$\Rightarrow \dfrac{340}{4L} = 250$ [v = speed of sound]

$$L = \frac{340}{4\times 250} = \frac{340}{1000} = 0.34m = 34\,cm$$

Length of shortest closed organ pipe is 34 cm

12.(7) $y_{net} = A_1\sin\theta + A_2\sin(\theta+\phi)$

$= (A_1 + A_2\cos\phi)\sin\theta + (A_2\sin\phi)\cos\theta = a\sin\theta + b\cos\theta$

$y_{net} = \sqrt{a^2+b^2}\sin(\theta+\beta)$ $\qquad$ $\beta = \tan^{-1}\dfrac{b}{a} = \tan^{-1}\dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}$

$A_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$ $\quad$ where $\phi = kx_0$ $\quad$ $\phi = 6.28\times 3.5 = 2\pi\times\dfrac{7}{2} = 7\pi$

$= |A_1 - A_2| = 12 - 5 = 7\ mm$

13.(10) Given,

$\mu = 9 \times 10^{-4}\ kg\ /\ m$

$T = 900\,N$

$\therefore\ V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{900}{9 \times 10^{-4}}} = 1000\,m\ /\ s$

Given, one of the resonating frequency

$f_n = 500\,\mu z$

$\therefore\ n \cdot \frac{V}{2l} = 500 \qquad \ldots$ (i)

and also,

$f_{n+1} = 550\,Hz$

So, $(n+1)\frac{V}{2l} = 550Hz \qquad \ldots$ (ii)

From (i) & (ii)

$\frac{n+1}{n} = \frac{550}{500}; \qquad n = 10$

Substitute the value in equation (i)

$10 \cdot \frac{V}{2l} = 500 \qquad \therefore\ l = \frac{5 \cdot V}{500}$

$l = \frac{V}{100} = \frac{1000}{100} = 10m$

14.(132) Apparent freq. heard by this person on the wall will be

$f_1 = \frac{v}{v - v_C}\,fo$

The same frequency will reflet back so freq. heard by the person is the car itself will be

$f\,' = \frac{v + v_c}{v} \times f_1 = \frac{v + v_c}{v - v_c} \times f_0$

$500 = \frac{330 + v_c}{330 - v_c} \times 400\ ; \quad \frac{330 + v_c}{300 - v_c} = \frac{5}{4}$

$v_c = \frac{110}{3}m\ /\sec \qquad = \frac{110}{3} \times \frac{18}{5} = 132\ km\ /\ hr$

So ans is 132 km/hr

15.(Bonus)

Given $Y = \frac{1}{(1+x)^2}$ at $t = 0$. If we put $x = -1$

We get $Y \to \infty$

So this does not represent wave.

Correct question should be $Y = \frac{1}{1 + x^2}$ at $t = 0$

$Y = \frac{1}{1 + (x-2)^2}$ at $t = 2\sec$

However NTA may claim amplitude to be at $x = 0$ at $t = 0$ and amplitude to be at $x = 2$ at $t = 1\sec$.
Even with given functions and hence ans 2m/s

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1.(AD) $f = f_s\left(\dfrac{v}{v-u}\right)$

(A) $u = 0.8v$ $\quad f = f_0\left(\dfrac{1}{0.2}\right) = 5f_0$

(B) $u = 0.8v$ $\quad f = 10f_0$

(C) $u = 0.8v$ $\quad f = \dfrac{5}{2}f_0$

(D) $u = 0.5v$ $\quad f = 3f_0$

Since the resonant frequencies in a one end open pipe are $f_0, 3f_0, 5f_0, \ldots\ldots.$

Hence resonance will be observed in A and D.

Solutions of Archive - JEE Main & Advanced

Wave Motion	Class - XI | Physics

JEE Main 2022

1.(A) $A_1 = 5$; $A_2 = 3$

$\Delta\theta = 2\pi(1.5) = 3\pi$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(3\pi)} = |A_1 - A_2| = 2cm$

2.(5) $A = 10\cos\left(\frac{4\pi}{3}\right) = 5cm$

3.(80) $2\left(\frac{V}{2L_1}\right) = \frac{V}{4L_2} \quad \Rightarrow \quad L_1 = 4L_2 = 80cm$

4.(152) $f_1 = f_0$; $\quad f_1, f_2, f_3, \ldots\ldots\ldots f_{20}$

$f_0, f_0 + 4, f_0 + 8, \ldots\ldots\ldots f_0 + 4 \times 19$

Given $\quad f_{20} = 2f_0$

$\Rightarrow \quad f_0 + 4\times 19 = 2f_0 \quad \Rightarrow \quad f_0 = 4\times 19 = 76 \quad \therefore \quad f_{20} = 2f_0 = 152\ Hz$

5.(A) Observer

$\rightarrow$. source

$f' = f\frac{\left(V + \frac{V}{5}\right)}{(V)} = \frac{6}{5}(f)$; %change $= \left(\frac{f' - f}{f}\right)(100) = \left(\frac{6}{5} - 1\right)(100) = 20\%$

6.(D) $f_{beat} = f_1 - f_2 = \frac{40}{12}$

$\Rightarrow \quad \left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = \frac{40}{12} \quad \Rightarrow \quad v\left[\frac{1}{4.08} - \frac{1}{4.16}\right] = \frac{40}{12}$

$\Rightarrow v = \frac{40}{12} \times \frac{4.08 \times 4.16}{0.08} = 707.2m / s$

7.(50) $V = v_\lambda$

$\lambda = \frac{V}{v} = \frac{340}{340} = 1\,meter$

First resonance length

$l_1 = \frac{\lambda}{4} = \frac{1}{4} = 0.25\,meter = 25cm$

Second resonance length

$l_2 = \frac{3\lambda}{4} = \frac{3\times 1}{4} = 0.75\,meter = 75cm$

Third resonating length

$l_3 = \frac{5\lambda}{4} = \frac{5\times 1}{4} = 1.25\,meter = 125cm$

The minimum height = 125 – 75 = 50 cm

8.(B) Maximum particle velcoity $v = A\omega$

and $v_{wave} = \frac{\omega}{k}$; $Aw = \frac{4w}{k} \Rightarrow \quad A = \frac{4}{k}$ *or* $\lambda = \frac{10}{4} \times 2\pi \quad = 5\pi$

9.(104) $\lambda = \frac{V}{\nu} = \frac{336}{400} = 84$ (in cm)

At first resonance ; $\frac{\lambda}{4} = \ell + e$; $\quad \frac{84}{4} = 20 + e$; $\quad e = 1$

$\therefore$ 3^{rd} resonance length

$5\frac{\lambda}{4} = \ell_2 + 1$; $\quad \frac{5(84)}{4} = \ell_2 + 1$; $\quad \ell_2 = 104\ cm$

10.(15) $v = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu v^2$

$$\Delta L = \frac{FL}{AY} = \frac{TL}{AY} = \frac{\mu v^2 L}{AY} = \frac{m}{L}\frac{v^2 L}{AY}$$

$$\Delta L = \frac{mv^2}{AY} = \frac{\left(10 \times 10^{-3}\right) \times 60 \times 60}{2 \times 10^{-6} \times 1.2 \times 10^{11}}$$

$$= \frac{6 \times 6}{2.4} \times 10^{-5}$$

$= 15 \times 10^{-5} m$ **;** $\quad x = 15$

11.(B) Given that

$$\sqrt{\frac{3RT}{M}} = \sqrt{2}\sqrt{\frac{\gamma RT}{M}}$$

$\gamma = \frac{3}{2}$; $\quad f = \frac{2}{y - 1} = 4$

Equivalent degree of freedom

$f = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$; $\quad 4 = \frac{2 \times 3 + 5n}{2 + n}$; $\quad n = 2$

12.(60) $A_R = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$

$$3A^2 = A^2 + A^2 + 2A^2 \cos\phi$$

$\frac{1}{2} = \cos\phi$; $\quad \phi = 60°$

13.(200) $\upsilon = \upsilon_0 \left(\frac{V \pm V_0}{V \mp Vs}\right)$ Doppler effect

$100 = \upsilon_0 \left(\frac{V}{V - V_c}\right) \Rightarrow$ car approaching ...(1)

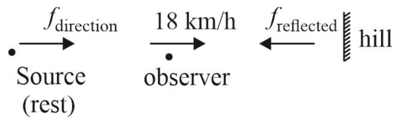
$50 = \upsilon_0 \left(\frac{V}{V + V_c}\right) \Rightarrow$ car receding ...(2)

$2 = \frac{V + V_c}{V - V_c}$; $\quad 2V - 2V_c = V + V_c$; $\quad V = 3V_c$

$\therefore \quad \upsilon_0 = \frac{50\left(V + V_c\right)}{V} \quad = \frac{50\left(4V_c\right)}{3V_c}$

$\upsilon_0 = \frac{200}{3} H_z$; $\quad \boxed{x = 200}$

14.(20) $F_{direct} = \left(\frac{320-5}{320}\right) f_0$



$= \frac{315}{320} \times 640 = 630\,Hz$

$f_{reflected} = \left(\frac{320+5}{320}\right) \times 640 \qquad = 650\,Hz$

Beat frequency $= |650 - 630| = 20\,Hz$

15.(A) $V_{\max} = a\omega = V_{wave}$

$\Rightarrow \qquad 2\omega = \frac{\omega}{k} \Rightarrow \qquad k = \frac{1}{2}$

$\frac{2\pi}{\lambda} = \frac{1}{2}; \qquad (\lambda = 4\pi)$

16.(3) $f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$

$f_{n+1} - f_n = \frac{(n+1)}{2l}\sqrt{\frac{T}{\mu}} - \frac{n}{2l}\sqrt{\frac{T}{\mu}} = 50 \qquad \Rightarrow \qquad \frac{1}{2l}\sqrt{\frac{T}{\mu}} = 50$

$\Rightarrow \qquad \mu = \frac{T}{4l^2} \times \frac{1}{(50)^2} = \frac{2700}{4 \times \left(30 \times 10^{-2}\right)^2 \times (50)^2} = 3\,kg/m$

17.(C) General wave equation can be written

As:- $y = y_0 \sin\left(\frac{2\pi}{T}t - \frac{2\pi x}{\lambda}\right)$

$y = y_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{T}t - x\right)\right]$

Comparing with given equation

$\frac{\lambda}{T} = 400\ m/s \qquad$ or $\qquad \lambda f = v = 400\ m/s$

18.(340) $f' = 320\left(\frac{330+10}{330-10}\right) = 340\,Hz$

19.(D) $f_{beat} = f_1 - f_2 = \frac{40}{12} \qquad \Rightarrow v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = \frac{40}{12}$

$\Rightarrow v\left[\frac{1}{4.08} - \frac{1}{4.16}\right] = \frac{40}{12}$

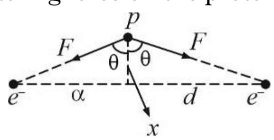
$\Rightarrow v = \frac{40}{12} \times \frac{4.08 \times 4.16}{0.08} = 707.2\,m/s$

Solutions of ARCHIVE - JEE Main & Advanced

Electrostatics	Class - XII | Physics

JEE Main 2021

1.(C) Net restoring force on the proton,

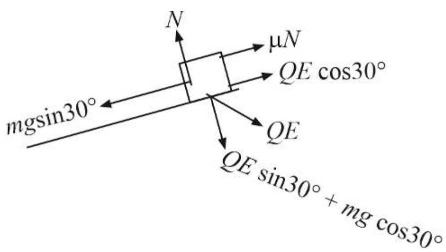


$F_{\text{net}} = 2F\cos\theta \qquad = \dfrac{2q^2}{4\pi\in(d^2+x^2)}\cdot\dfrac{x}{(d^2+x^2)^{1/2}}$

As $x << d$, $\qquad F_{\text{net}} \approx \left[\dfrac{q^2}{2\pi\in_0 d^3}\right]x$

Therefore, the angular frequency, $w = \left(\dfrac{q^2}{2\pi\in_0 md^3}\right)^{1/2}$

2.(A)



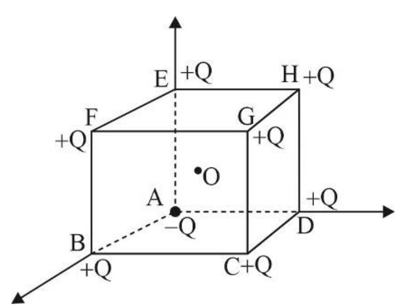
$$a = \frac{mg\sin\theta - QE\cos\theta - \mu(QE\sin\theta + mg\cos\theta)}{m}$$

3.(A) As we know, charges are symmetrically placed about the centre of cube. Electric field due to most of the charges will cancel out each other for eg: $\left[\vec{E}_B + \vec{E}_H = 0,\ \vec{E}_D + \vec{E}_F = 0,\ \vec{E}_C + \vec{E}_E = 0,\right]$

Since charges at A and G are of opposite sign their electric field will not get cancelled.

E_{net} at $O = \vec{E}_A + \vec{E}_G$ (where O is centre)

$\left|\vec{E}_{net}\right| = 2\times\dfrac{1}{4\pi\varepsilon_0}\times\dfrac{Q}{\left(\dfrac{a\sqrt{3}}{2}\right)^2}$ $\qquad \left(\left|\vec{E}_A\right| = \left|\vec{E}_G\right|\right)$



$$= \frac{2Q}{3\pi\varepsilon_0 a^2}$$

Direction of $\vec{E}_{net}$ will be towards $\overrightarrow{OA}$

Unit vector along $\overrightarrow{OA} = -\dfrac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$ or $\dfrac{-}{\sqrt{3}}(\hat{x}+\hat{y}+\hat{z})$

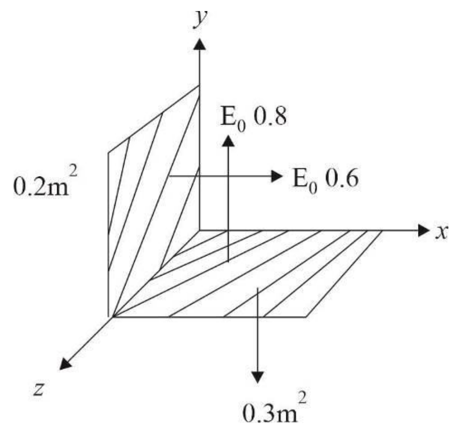
$$\vec{E}_{net} = \frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$$

4.(226) We can add 5 more squares around the point charge to aomplete a cube. Because of the symmetry of the situation, the flux through each face of the cube is $\frac{Q}{6\epsilon_0}$

So, $\phi = \frac{Q}{6\epsilon_0} = \frac{12\times10^{-6}}{6}\times 4\pi(9\times10^9)$

$= 72\pi\times10^3 = 72(3.14)\times10^3 = 226.08\times10^3\ Nm^2/C$

5.(1) $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)N/C$



$\frac{\phi_{yz}}{\phi_{xz}} = \frac{0.2\times E_0\times0.6}{0.3\times E_0\times0.8} = \frac{2}{3}\times\frac{6}{8} = 0.50$

$a : b = 1 : 2$

$a = 1$

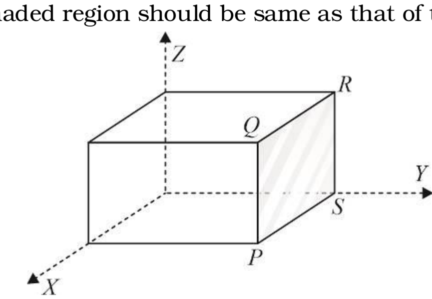
6.(128) $\frac{kq}{r} = 2$; $\frac{kQ}{R} = V$

Volume = constant

$\Rightarrow \frac{4\pi}{3}R^3 = n\frac{4\pi}{3}r^3 \Rightarrow R = (512)^{1/3}r$

$\therefore R = 8r \qquad \therefore V = \frac{k(512q)}{8r} = 64\times2 = 128 volt.$

7.(D) Flux associated with the shaded region should be same as that of the flux associated with area $PQRS$.



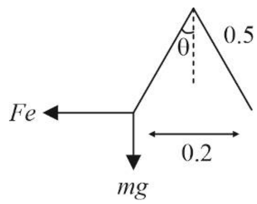
If we put seven more identical cubes so that the charge can be assumed at center of the bigger cube.

$\phi_{PQRS} = \frac{q}{24\varepsilon_0}$

8.(36) Because they were identical so charge on each sphere will be 1 nC after contact.

So, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{9\times10^9\times1\times10^{-18}}{(0.5)^2} = 36\times10^{-9}N$

9.(20)

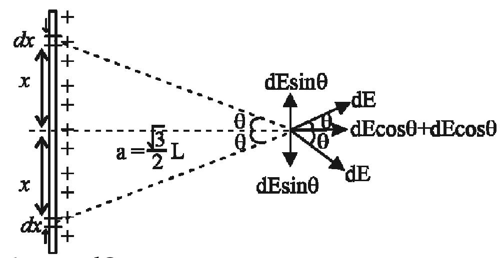


From diagram $Fe = mg\tan\theta$

$$\frac{9\times10^9\times q^2}{(0.2)^2} = 10\times10^{-6}\times10\times\frac{0.1}{0.5}; \qquad a = 20$$

10.(B) Electric field due to element charge length dx;

$$dE = \frac{1}{4\pi\epsilon_0}\frac{(dQ)}{(a^2+x^2)}$$



Component of electric field due to elements of wire along the perpendicular bisector will get added to give resultant electric field.

$$\therefore \qquad E_{net} = \int_{-L/2}^{L/2} dE\cos\theta = 2\int_0^{L/2} dE\cos\theta = 2\int_0^{L/2}\frac{1}{4\pi\epsilon_0}\frac{dQ}{(a^2+x^2)}\cdot\frac{a}{\sqrt{a^2+x^2}}$$

$$= \int_0^{L/2} 2\times\frac{1}{4\pi\epsilon_0}\left(\frac{Q}{L}dx\right)\frac{a}{(a^2+x^2)^{3/2}} = \frac{2}{4\pi\epsilon_0}\frac{Q}{L}a\left[\frac{1}{a^2}\cdot\frac{x}{\sqrt{x^2+a^2}}\right]_0^{L/2} = \frac{Q}{2\sqrt{3}\pi\epsilon_0 L^2}$$

11.(A) Conceptual

12.(243) Let charge on one small drop be = q

(radius r)

$$\therefore \qquad \frac{kq}{r} = 10\ V \qquad \text{(given)} \qquad \text{[metal mercury]}$$

Then radius of one large sphere $\Rightarrow$ $27\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\Rightarrow \qquad R = 3r$

and total charge on bigger sphere = $27q = Q$

$$U_1 = PE \text{ of metallic charged sphere } = \frac{KQ^2}{2R} \Rightarrow \frac{U_1}{U_2} = \left(\frac{Q}{q}\right)^2\frac{r}{R}$$

$$U_2 = PE \text{ of metallic small sphere } = \frac{Kq^2}{2r} = (27)^2\times\frac{1}{3} = 243$$

13.(640) $Q = E_x . A = \frac{2}{5}\times4\times10^3\times0.4 = 640\ Nm^2C^{-1}$

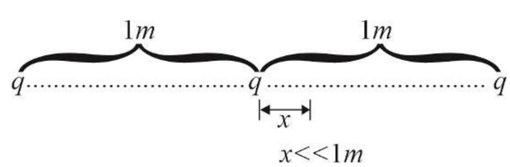
14.(12) $F = \frac{kq_1q_2}{r_1^2}+\frac{kq_1q_3}{r_2^2}+\frac{kq_1q_4}{r_3^2}+.......\infty$ (where $q_1 = 1C$ and all other charges are $1\ \mu C$)

$$= 9\times10^9\times10^{-6}\left(1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+.......\infty\right) = 9\times10^3\left(\frac{4}{3}\right) = 12\times10^3 N$$

$\therefore \ x = 12$

15. (Q. NOT clear)

16.(6000) $F = \frac{kq^2}{(1+x)^2} - \frac{kq^2}{(1-x)^2}$



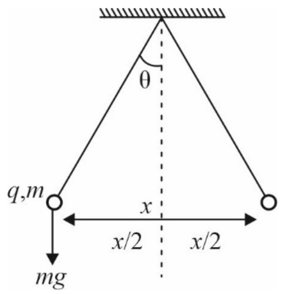
$$a=\frac{F}{m}=\frac{kq^2}{m}\left(\frac{1}{(1+x)^2}-\frac{1}{(1-x)^2}\right)=\frac{-4kq^2}{m}x$$

$$w=\sqrt{\frac{4kq^2}{m}}=\sqrt{\frac{4\times9\times10^9\times10}{10^{-6}}}=2\times3\times10^8=6000\times10^5\,rad/s$$

17.(B) $E_z=\frac{2P_1}{r^3}$ $\qquad E_y=\frac{P_2}{r^3}$

$\therefore \qquad \tan 37^\circ=\frac{E_y}{E_x}=\frac{P_2}{2P_1}=\frac{3}{4} \qquad \Rightarrow \qquad \frac{P_1}{P_2}=\frac{2}{3}$

18.(C) $\tan\theta=\frac{F_e}{mg}$



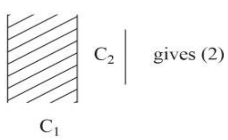
$Fe=\frac{Kq^2}{x^2}$

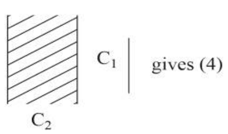
$\tan\theta=\frac{Kq^2}{x^2mg}\approx\sin\theta$

$\frac{x}{2l}=\frac{Kq^2}{x^2mg}$

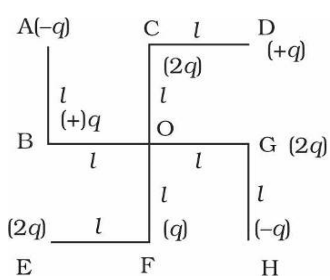
$x=\left(\frac{Kq^2\times2l}{mg}\right)^{1/3} \qquad \Rightarrow \qquad x=\left(\frac{q^2\times l}{2\pi\varepsilon_0 mg}\right)^{1/3}$

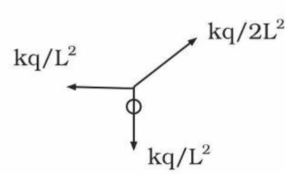
19.(D) Question has not satisfied which part is C_1 and C_2

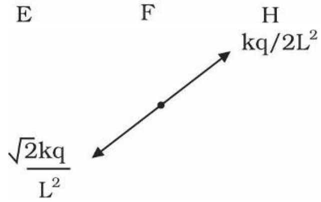




20.(A)



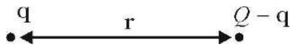




Net E field $=\frac{kq^2}{l^2}\left(\sqrt{2}-\frac{1}{2}\right)$

$=\frac{kq^2}{2L^2}\left(2\sqrt{2}-1\right)$

21.(A)



$$F = \frac{Kq_1q_2}{r^2} = \frac{Kq(Q-q)}{r^2}$$

$$\frac{dF}{dq} = 0$$

$$\frac{d}{dq}\left(q(Q-q)\right) = 0$$

$$q(0-1)+(Q-q)1 = 0$$

$$-q+Q-q = 0$$

$$\theta = 2q = 0$$

$$q = \frac{Q}{2}$$

$$Q = 2q$$

22.(1) $\frac{q}{m} = 8\mu c / gm = \frac{8\times10^{-6}C}{10^{-3}kg} = 8\times10^{-3}C/kg$

Time taken by the body to reach the wall

$$t = \sqrt{\frac{2d}{qE/m}} = \sqrt{\frac{2\times.1}{8\times10^{-3}\times100}}$$

$$t = \frac{1}{2}\sec$$

Same time taken by the body to return to the position form where it was released so time period $T = 2t$ $= 1\sec$

23.(D) $F = q\left[\frac{2\times9\times10^9\times3\times10^{-6}}{10\times10^{-3}} - \frac{2\times9\times10^9\times3\times10^{-6}}{12\times10^{-3}}\right]$

$$4 = q\ \ 2\times9\times10^6\times3\left[\frac{1}{10}-\frac{1}{12}\right]$$

$$4 = q\times2\times9\times3\times10^6\times\frac{1}{60}$$

$$q = \frac{40}{9}\times10^{-6}C = 4.44\mu C$$

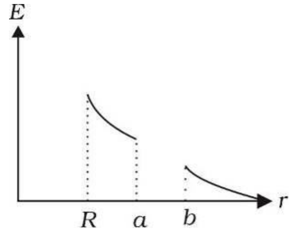
24.(4) $\vec{\Delta}.\vec{D} = \rho$

$$\rho = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right).\left(e^{-x}\sin y\hat{i} - e^{-x}\cos y\hat{j} + 2z\hat{k}\right)$$

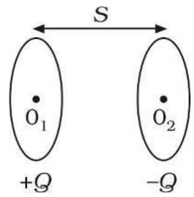
$$= 2$$

$$q = 2\times2\times10^{-9} = 4\ nC$$

25.(D) Considering outer shell as conducting, graph will be as shown. Best possible match is option -4



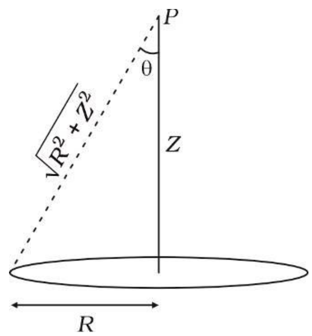
26.(D)



$$V_{0_1} = \frac{KQ}{a} - \frac{KQ}{\sqrt{S^2 + a^2}} \text{ and } V_{0_2} = -\frac{KQ}{a} + \frac{KQ}{\sqrt{S^2 + a^2}}$$

$$\Delta V = V_{0_1} - V_{0_2} = \frac{2KQ}{a} - \frac{2KQ}{\sqrt{S^2 + a^2}} = \frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{\sqrt{S^2 + a^2}}\right]$$

27.(C)

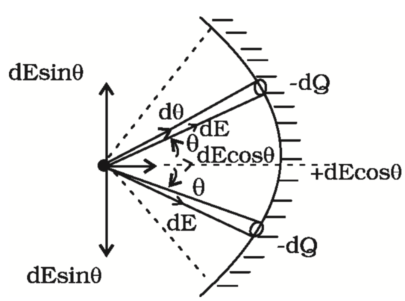


Field at point P,

$$E = \frac{\sigma}{2\in_0}\left(1 - \cos\theta\right)$$

$$= \frac{\sigma}{2\in_0}\left(1 - \frac{Z}{\sqrt{R^2 + Z^2}}\right)$$

28.(D)



Electric field due to small element charge at an angle θ both sides from bisector line;

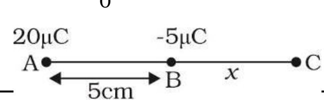
$$dE = \frac{k\left(dQ\right)}{R^2} = \frac{1}{4\pi\in_0}\frac{Q}{\left(\frac{2\pi}{3}.R\right)}\frac{Rd\theta}{R^2}$$

Component perpendicular to bisector line will get cancelled.

$$E_{net} = \int_{\theta=0}^{\theta=\frac{\pi}{3}} 2dE\cos\theta = 2\times\int_{\theta=0}^{\frac{\pi}{3}}\frac{1}{4\pi\in_0}\frac{Q}{\frac{2\pi}{3}R}\frac{\left(Rd\theta\right)}{R^2}\times\cos\theta$$

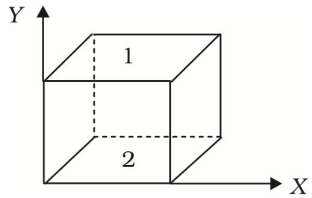
$$= \frac{3Q}{4\pi^2\in_0 R^2}\left[\sin\theta\right]_0^{\pi/3} = \frac{3\sqrt{3}}{8}\frac{Q}{\pi^2\in_0 R^2}$$

29.(B)



$$\frac{Kq_1}{r_1^2} = \frac{Kq_2}{r_2^2} \Rightarrow \frac{20}{(5+x)^2} = \frac{5}{x^2} \quad \Rightarrow 2x = 5 + x \Rightarrow x = 5$$

30.(D)



$\phi = \phi_1 + \phi_2$, flux theough rest of

$\downarrow$ the surface = 0

0 as $\vec{E} = 150y^2\hat{j}$

$$\frac{q}{\varepsilon_0} = 150 \times (0.5)^2 \times (0.5)^2$$

$$q = 8.2 \times 10^{-11} C$$

31.(D)

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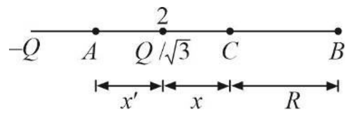
1.(1.73)2.(3)



V at B:

$$\frac{KQ}{(2+R+x)} = \frac{\frac{KQ}{\sqrt{3}}}{x+R}$$

$$\sqrt{3}(x+R) = 2+R+x$$

$$(\sqrt{3}-1)x + (\sqrt{3}-1)R = 2 \qquad \ldots \text{(i)}$$

V at A:

$$\frac{KQ}{2-x'} = \frac{\frac{KQ}{\sqrt{3}}}{x'}; \qquad \sqrt{3}x' = 2 - x'$$

$$x' = \frac{2}{\sqrt{3}+1}; \qquad x' + x = R$$

$$\frac{2}{\sqrt{3}+1} + x = R$$

$$2 + (\sqrt{3}+1)x = (\sqrt{3}+1)R$$

$$x = \frac{(\sqrt{3}+1)R - 2}{\sqrt{3}+1}$$

$$(\sqrt{3}+1)R - (\sqrt{3}+1)x = R \qquad \ldots \text{(ii)}$$

Using equation (i) and (ii) $\qquad R = \sqrt{3}\ m$

$$x = \frac{(\sqrt{3}+1)\sqrt{3}-2}{\sqrt{3}+1} = \frac{\sqrt{3}+1}{\sqrt{3}+1} = 1\ m \ \Rightarrow \qquad b = 3$$

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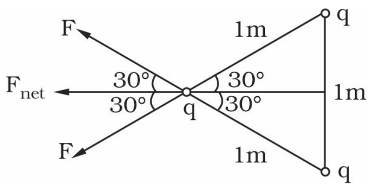
1.(D)



$$F = \frac{k(q)(q)}{1^2} = \frac{kq^2}{1}$$

$$F_{net} = 2 \times F \times \cos 30°$$

$$= \cancel{2}\, kq^2 \left(\frac{\sqrt{3}}{\cancel{2}}\right)$$

$$= \frac{F_{net}}{F} = \frac{kq^2}{kq^2} = \left(\frac{\sqrt{3}}{1}\right)$$

2.(B) By gauss's theorem,

$$(\phi_E)total = \frac{q}{\epsilon_0}$$

$$(\phi_E)cs + (\phi_E)_{flat} = \frac{q}{\epsilon_0}$$

$$E\left(2\pi R^2\right) + (\phi_E)_{flat} = \frac{q}{\epsilon_0}$$

$$\left(\frac{1}{4\pi \epsilon_0} \times \frac{Q}{R^2} \times 2\pi R^2\right) + (\phi_E)_{flat} = \frac{q}{\epsilon_0}$$

$$\frac{q}{2 \epsilon_0} + (\phi_E)_{flat} = \frac{q}{\epsilon_0}$$

$$(\phi_E)_{flat} = \frac{q}{2 \epsilon_0}$$

3.(B) $mg = qE$

$$(0.1 \times 10^{-3})(9.8) = 4.9 \times 10^5 q$$

$$\frac{2 \times 10^{-4}}{10^5} = q$$

$$q = 2 \times 10^{-9} C$$

4.(A) For the charged particles to remain in equilibrium, the electrostatic force should be equal to frictional force

$\Rightarrow \dfrac{Kq^2}{L^2} = \mu mg$

$\Rightarrow L = q\sqrt{\dfrac{K}{\mu mg}}$

$= 2\times10^{-7}\sqrt{\dfrac{9\times10^9}{0.25\times10\times10^{-3}\times10}} = 12\times10^{-2}m = 12\,cm$

5.(A) $E\times 2\pi r\ell = \dfrac{\rho\pi R^2\ell}{\varepsilon_0}$ {by gauss' law}

$E = \dfrac{\rho R^2}{2\varepsilon_0 r}$

$\dfrac{mv^2}{r} = qE = \dfrac{\rho R^2 q}{2\varepsilon_0 r}$

$\dfrac{1}{2}mv^2 = \dfrac{\rho R^2 q}{4\varepsilon_0}$

6.(C) $E = \dfrac{\sigma}{2\varepsilon_0}$ is independent of distance from the plane sheet.

7.(198) Let each small drop has charge q and radius R.

$\dfrac{kq}{R} = 22V$ ...(i)

When all the drops combine, volume and total charge are conserved.

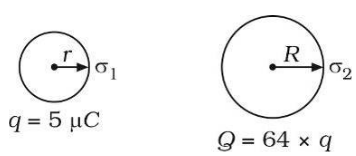
$27\left[\dfrac{4}{3}\pi R^3\right] = \dfrac{4}{3}\pi R'^3$

$\Rightarrow R' = 3R$

Also, $q' = 27q$

Thus, $V' = \dfrac{kq'}{R'} = \dfrac{k(27q)}{3R}$ $\Rightarrow V' = \dfrac{9kq}{R} = 9\times22 = 198V$

8.(B)



$64\times\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

$\Rightarrow$ $R = 4\times r$

$\therefore$ $\dfrac{\sigma_2}{\sigma_1} = \dfrac{Q}{4\pi R^2}\times\dfrac{4\pi r^2}{q} = 4:1$

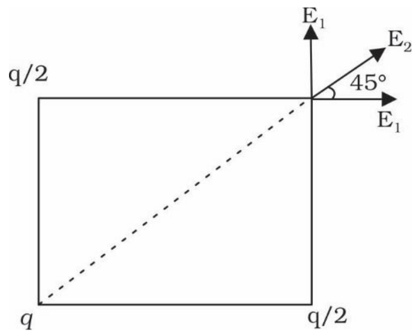
9.(B) There can be cases where net force on a dipole in non-uniform electric field may be zero.

$F = -\rho\dfrac{\partial E}{\partial \ell}$

E.g. Where there is a maxima of electric $\dfrac{\partial E}{\partial L}=0 \Rightarrow F = 0$ field.

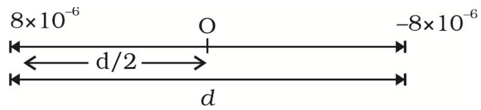
Correct Option is B

10.(A)



$$E_{net} = \sqrt{2}\left(\frac{1}{4\pi \in_0}\frac{q/2}{a^2}\right) + \frac{1}{4\pi \in_0}\frac{q}{(\sqrt{2}a)^2} = \frac{1}{4\pi \in_0}\frac{q}{a^2}\left[\frac{\sqrt{2}}{2} + \frac{1}{2}\right]$$

11.(B) $E_0 = \dfrac{9\times 10^9 \times \left(8\times 10^{-6}\right)\times 2}{\dfrac{d^2}{4}}$



$$\Rightarrow \qquad E_0 = \frac{576\times 10^3}{d^2}$$

$$\Rightarrow \qquad 6.4\times 10^4 = \frac{576\times 10^3}{d^2}$$

$$\Rightarrow \qquad d^2 = \frac{90}{10}$$

$d = \pm 3m$

12.(D) By Work Energy Theorem :

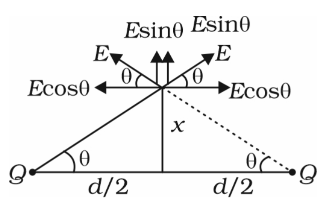
$\Rightarrow \qquad W_E = \Delta K$

$$-qE\times(\text{displacement}) = 0 - \frac{1}{2}mv^2$$

$$\text{displacement (d)} = \frac{mv^2}{2qE} = \frac{100\times 10^{-6}\times 200\times 200}{2\times 40\times 10^{-6}\times 10^5}$$

$$= 0.5m$$

13.(D)



$$F_{net} = (2E\sin\theta)q$$

$$= 2\frac{kQ}{\left(\frac{d}{2}\right)^2 + x^2}\times\frac{x}{\sqrt{\left(\frac{d}{2}\right)^2 + x^2}}q$$

$$F_{net} = 2kQq\frac{x}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$$

For maxima $\frac{dF_{net}}{dx} = 0$

$$\frac{\left(x^2 + \frac{d^2}{4}\right)^{3/2}(1) - \left(\frac{3}{x}\left(x^2 + \frac{d^2}{4}\right)^{3/2^{-1}} \times 2x\right)x}{\left(x^2 + \frac{d^2}{4}\right)^3} = 0 \; ; \qquad \left(x^2 + \frac{d^2}{4}\right) = 3x^2$$

$$2x^2 = \frac{d^2}{4}; \qquad x = \frac{d}{2\sqrt{2}}$$

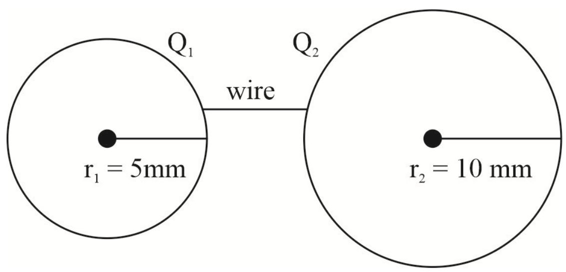
14.(D) $E = \frac{-\partial v}{\partial r}$; $\quad E = \frac{-\partial v}{dx} = \frac{-d}{dx}(3x^2)$

$E = -3(2x) = -6x$

At point (1, 0, 3)

$E = -6(1) = -6Vm^{-1} \quad \therefore \quad E = 6Vm^{-1}$ along $-x$ axis

15.(B)



As they are connected by an conducting wire potential of Both spheres is same

$V_1 = V_2$

$\frac{kQ_1}{r_1} = \frac{kQ_2}{r_2} \qquad ...(1)$

Now ratio of Electric fields on surface

$\frac{E_1}{E_2} = \frac{\frac{kQ_1}{r_1^2}}{\frac{kQ_2}{r_2^2}} = \frac{r_2}{r_1} = \frac{10}{5} = \frac{2}{1}$ (for mutual interaction to be ignored value of d should be large)

$E_1 : E_2 \Rightarrow 2:1$

16.(45) No. of electric field lines per unit area = electric field

For uniformly charged solid sphere,

$E = \frac{\rho r}{3\varepsilon_0}$

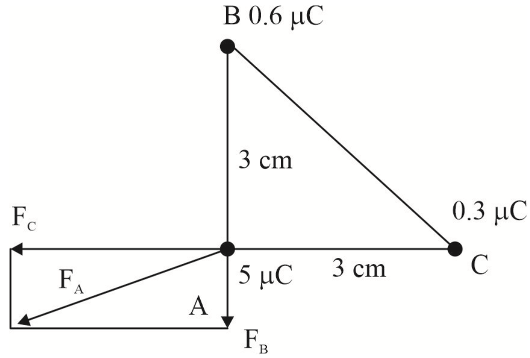
At surface, $r = R$

$E = \frac{\rho R}{3\varepsilon_0}$

$E = \frac{\rho R}{3\varepsilon_0} = \frac{2 \times 6}{3 \times 8.85 \times 10^{-12}} = 0.45 \times 10^{-12} NC^{-1}$

$= 45 \times 10^{10} N/C$

17.(17) Net electrostatic froce on A in the resultant of F_C & F_B



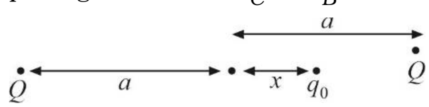
$$F_A = \sqrt{F_C^2 + F_B^2}$$

$$F_C = \frac{9\times 10^0 q_C q_A}{(CA)^2}$$

$$F_B = \frac{9\times 10^9 \times q_B q_A}{(BA)^2}$$

putting the value of F_C & F_B and solving we get $F_A = 17N$

18.(A)



$$F_{net} = \frac{KQq_0}{(a-x)^2} - \frac{KQq_0}{(a+x)^2}$$

$$F_{net} = \frac{KQq_0}{a^2}\left\{\left(1-\frac{x}{a}\right)^{-2} - \left(1+\frac{x}{a}\right)^{-2}\right\} \quad ; \quad F_{net} = \frac{KQq_0}{a^2}\left\{1+\frac{2x}{a} - 1 + \frac{2x}{a}\right\}$$

$$m\omega^2 x = \frac{1}{4\pi\varepsilon_0}\times\frac{Qq_0}{a^2}\times\frac{4x}{a} \quad ; \quad \omega = \sqrt{\frac{Qq_0}{\pi\varepsilon_0 m a^3}}$$

$$T = \frac{2\pi}{\omega} = \sqrt{\frac{4\pi^3\varepsilon_0 m a^3}{Qq_0}}$$

19.(1) Electric field inside along cylindrical volume having uniform charge distribution is $= \dfrac{\rho r}{2\in_0}$

$$E = \frac{\rho}{2\in_0}\times\frac{2\in_0}{\rho} = 1$$

20.(B) $q_1 + q_2 = 4$

$$F = \frac{kq_1q_2}{r^2}$$

$$q_1 = q_2 = 2\mu F$$

21.(6) $\dfrac{\tau_1}{\tau_2} = \dfrac{P_1E_1}{P_2E_2} = \dfrac{1}{6}$

22.(B) $a_y = \dfrac{eE}{m}$, t time to cross $= \dfrac{\ell}{v} = \dfrac{1}{2} = 0.5\,\text{sec}$

$V_y = a_y t,\ V_x = 2m/s$

$$\tan\theta = \frac{V_y}{V_x} = \frac{eEt}{m \times 2} = \frac{e\left(\frac{8m}{e}\right) \times 0.5}{m \times 2} = 2\,; \qquad \theta = \tan^{-1} 2$$

23.(C) Using gauss Law : Considering Gaussian surface of radius x.

$$\oint \vec{E} \cdot \overrightarrow{dA} = \frac{\int_0^x 4\pi r^2 \rho(r)\, dr}{\varepsilon_0}$$

$$4\pi x^2 E = \frac{4\pi\rho_0}{\varepsilon_0} \int_0^x r^2 \left(\frac{3}{4} - \frac{r}{R}\right) dr = \frac{4\pi\rho_0}{\varepsilon_0}\left[\frac{x^3}{4} - \frac{x^4}{4R}\right]$$

$$4\pi x^2 \times E = \frac{4\pi\rho_0 x^3}{4\varepsilon_0}\left[1 - \frac{x}{R}\right]$$

$$E(x) = \frac{\rho_0 x}{4\varepsilon_0}\left[1 - \frac{x}{R}\right] \quad ; \quad E(r) = \frac{\rho_0 r}{4\varepsilon_0}\left[1 - \frac{r}{R}\right]$$

24.(A) Both the statements are correct.

25.(B) Let charge be q on both sphere and distance be r

$$F = \frac{kqq}{r^2}$$

Further, C is placed in contact of A,

So, charge on each will be $\frac{q}{2}$.

Then C is then placed in contact with B

So, new charge on C and B will be $\frac{\frac{q}{2} + q}{2} = \frac{3q}{4}$

Now $A\left(\frac{q}{2}\right)$ $\qquad C\left(\frac{3q}{4}\right)$ $\qquad B\left(\frac{3q}{4}\right)$

$$\frac{k\left(\frac{q}{2}\right)\left(\frac{3q}{4}\right)}{\left(\frac{r}{2}\right)^2} \quad \Bigg| \quad \frac{k\left(\frac{3q}{4}\right)\left(\frac{3q}{4}\right)}{\left(\frac{r}{2}\right)^2}$$

$$= \frac{3kq^2}{8} \times \frac{4}{r^2} \qquad = \frac{9kq^2}{16} \times \frac{4}{r^2}$$

$$= \frac{3F}{2} \qquad = \frac{9F}{4}$$

$\therefore$ $\quad Fnet = \frac{9F}{4} - \frac{3F}{2} = \frac{3F}{4}$

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Solution of Archive - JEE Main & Advanced

DC Circuits	Class - XII | Physics

JEE Main 2021

1.(A) $i = \alpha_0 t + \beta t^2$

$$\Rightarrow \frac{dq}{dt} = \alpha_0 t + \beta t^2$$

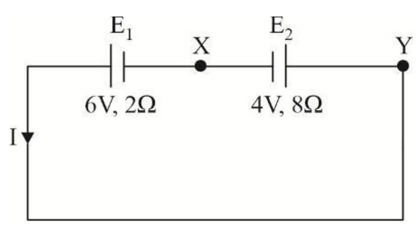
$$\Rightarrow q = \int_0^{15} (\alpha_0 t + \beta t^2)\, dt$$

$$\Rightarrow q = \left[\alpha_0 \frac{t^2}{2} + \beta \frac{t^3}{3}\right]_0^{15}$$

$$= 20 \times \frac{(15)^2}{2} + 8 \times \frac{(15)^3}{3}$$

$$= 11250\ C$$

2.(D) Current in the circuit is $I = \frac{6-4}{10} = \frac{1}{5} A$



Applying K.V.L.,

$$V_x + E_2 + 8I = V_y$$

$$V_x - V_y = -(E_2 + 8I)$$

$$V_x - V_y = -(4 + 1.6)V$$

$$V_x - V_y = -5.6\ V$$

3.(5) Current, $i = \frac{v}{R}$

$$= \frac{El}{\left(\frac{\rho l}{\pi r^2}\right)} = \pi\left(\frac{Er^2}{\rho}\right)$$

$$= \pi(E\sigma r^2)$$

So, $i = \pi(10 \times 10^{-3})\,(5 \times 10^7)\,(0.5 \times 10^{-3})^2$

$$= 125\pi \times 10^{-3} A$$

$$= 125\pi\ mA$$

$$= 5^3 \pi\ mA$$

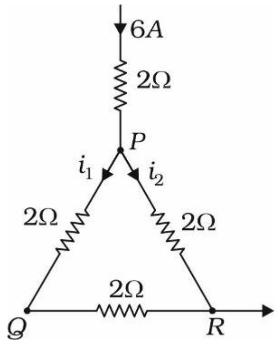
4.(Bonus/1) If polarity of known battery is changed, the answer is 1

$$\frac{E_1}{E_2} = \frac{\phi 380}{\phi 760} = \frac{1}{2}$$

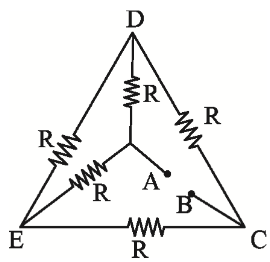
$a = 1$

Otherwise, no Null point would come.

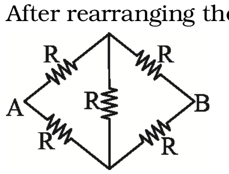
5.(2) $\frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{2}{4}$ & $i_1 + i_2 = 6$ $\Rightarrow$ $i_1 = 2A$



6.(C)



After rearranging the circuit



Wheat stone bridge circuit

So, $R_{AB} = R$

7.(300) Work done by battery $= QV = 20 \times 15 = 300J$

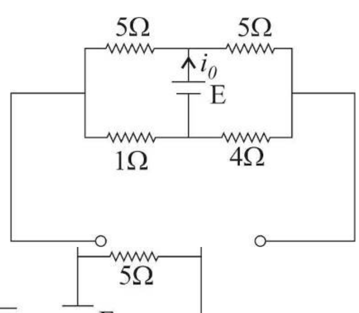
8.(A) $R = \frac{\rho L}{A}$

$\text{Volume} = L \times A = 1.25L \times \frac{A}{1.25}$

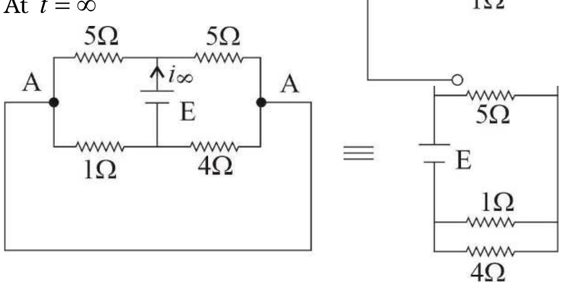
$1.25 \times 1.25 = 1.5625$

9.(B) At t = 0

$i_0 = \frac{E}{\frac{6 \times 9}{6+9}} = \frac{15E}{54} = \frac{5E}{18}$



At $t = \infty$



$$i_\infty = \frac{E}{\frac{5}{2}+\frac{4}{5}} = \frac{10E}{33}$$

10.(A) $I = neAv_d$

$\Rightarrow \quad 10 = n \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 2 \times 10^{-3}$

$\Rightarrow \quad n = \dfrac{10^{25}}{1.6 \times 10^{-3}} \quad \therefore \quad n = 625 \times 10^{25}$

11.(4) $s = r_1 + r_2$

$p = \dfrac{r_1 r_2}{r_1 + r_2}$

$s = np \Rightarrow (r_1 + r_2)^2 = n r_1 r_2$

$n = \dfrac{r_1}{r_2} + \dfrac{r_2}{r_1} + 2$

For n to be max, $r_1 = r_2 \Rightarrow n = 4$

12.(D) $R = \rho \dfrac{l}{A}$ also, $V = iR$

Now, $l' = 2l$

$A' = \dfrac{A}{2}$

$R' = \rho \times \dfrac{2l}{\frac{A}{2}} = 4\dfrac{\rho l}{A} = 4R \quad \therefore \quad i' = \dfrac{V}{R'} = \dfrac{V}{4R} = \dfrac{VA}{4\rho l}$

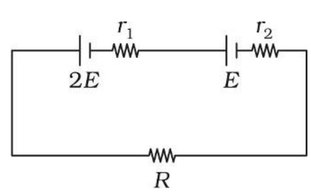
13.(3) Net resistance in the circuit is $7k\Omega$

So, current $= \dfrac{21}{7} \times 10^{-3} A$

= 3 mA

This current will pass through $5k\Omega$ resistance

14.(B) $i = \dfrac{3E}{R + r_1 + r_2}$



$V_1 = 2E - i\,r_1 = 0$

$2E - \dfrac{3E}{R + r_1 + r_2} \cdot r_1 = 0$

$2R + 2r_1 + 2r_2 - 3r_1 = 0$

$R = \dfrac{r_1}{2} - r_2$

15.(70) Let the voltage required be x

Then the current through battery = $x/10$

R_{eq} of parallel combination = $\dfrac{100}{7}\Omega$

Thus applying KVL we get

$x + \dfrac{100}{7} \times \dfrac{x}{10} = 170$

$x \times \dfrac{17}{7} = 170 \quad \Rightarrow \quad x = 70\ V$

16.(48) As we know,

$\dfrac{12}{x} = \dfrac{6}{72 - x} \quad \therefore \quad \dfrac{2}{x} = \dfrac{1}{72 - x} \quad \therefore \quad x = 48$

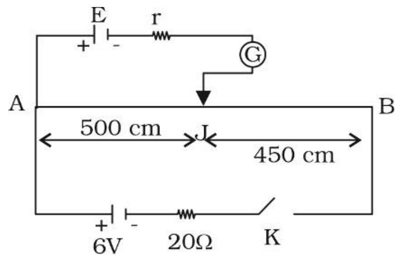
17.(4) In parallel combination,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\rho_{eq}\frac{l}{2A} = \frac{\rho_1 \frac{l}{A} \times \rho_2 \frac{l}{A}}{\frac{l}{A}(\rho_1 + \rho_2)} = \left(\frac{6 \times 3}{6+3}\right)\frac{l}{A} \qquad \therefore\ \rho_{eq} = 4$$

18.(10) $R_{ab} = \frac{12 \times 6}{12+6} + \frac{4}{2} + \frac{6 \times 14}{6+12} = 4\Omega + 2\Omega + 4\Omega = 10\Omega$

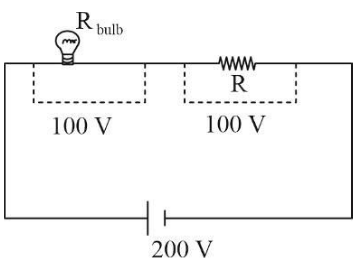
19.(A) Maximum emf is potential difference across AB.



$\Rightarrow R_{AB} = 0.1 \times 1000 = 100\Omega$

$\Rightarrow I$ by battery of emf $6V = \frac{6}{20+100} = \frac{6}{120}$ $\qquad \Rightarrow \Delta V_{AB} = R_{AB} \times I = 100 \times \frac{6}{120} = 5\,volts$

20.(50) $R_{bulb} = \frac{V^2}{P} = \frac{(100)^2}{200}$



The bulb will give the same power when potential difference across it becomes 100 V.

$\therefore V_R = V_{bulb} = \frac{200}{2} = 100\,volts \qquad \therefore R = R_{bulb} = \frac{V^2}{P} = \frac{(100)^2}{200} = 50\Omega$;

21.(D) Since balancing length $\propto$ emf

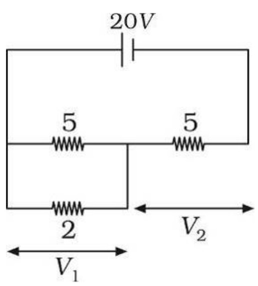
$\varepsilon_1 \propto 250$

$\varepsilon_1 + \varepsilon_2 \propto 400$

$\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1} = \frac{16}{10} = \frac{8}{5}$

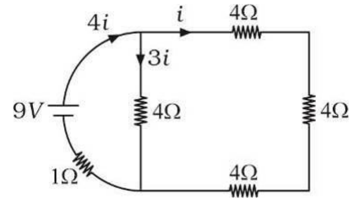
$\frac{\varepsilon_2}{\varepsilon_1} = \frac{3}{5}$ or $\frac{\varepsilon_1}{\varepsilon_2} = \frac{5}{3}$

22.(D)



$V_1 = \frac{10/7}{10/7+5}\times 20 \quad = \frac{10}{45}\times 20 = \frac{40}{9}$ volts

23.(45) here assume current as



By KVL in outer loop

$9 - 12i - 4i = 0$

$16i = 9$

$8i = \frac{9}{2} = 4.5 \quad = 45\times 10^{-1}$

24.(C) The current will remain the same in all sections

$V_\alpha = \frac{i}{neA}$

$V_\alpha \propto \frac{1}{A}$

As A is decreasing

$V\alpha$ is increasing

Also $E = \frac{dV}{dl}$

But by ohm's low

$dV = idR$

$dV = i\times\rho\frac{dl}{dA}$

$E = \frac{dV}{dl} = \frac{i\rho}{dA} \quad \Rightarrow \quad E \propto \frac{1}{dA}$

Electric field increases

25.(D) $16 = R_0(1+\alpha(15))$

$20 = R_0(1+\alpha(100))$

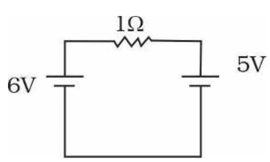
$\frac{5}{4} = \frac{1+\alpha(100)}{1+\alpha(15)}$

$5 + 75\alpha = 4 + 400\alpha$

$1 = 325\alpha$

$\alpha = .003 / °C$

26,(1) At $t = 3.2$ seconds



$i = \frac{6-5}{1} = 1A$

27.(C) $I = \vec{J}.\vec{A} = JA\cos 60°$ Gravi

$\Rightarrow \quad 5 = J\times 0.04\times\frac{1}{2} \Rightarrow J = 250\, A/m^2$

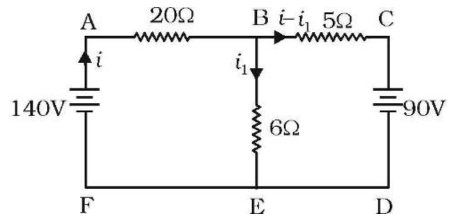
Also, $J = \frac{E}{\rho}$

$\Rightarrow \quad E = J\rho$

$= 250\times 44\times 10^{-8} = 11000\times 10^{-8}$

$= 11\times 10^{-5} V / M$

28.(B)



Applying KVL in loop $FABEF$

$140 - 20i - 6i_1 = 0$

or $\quad 10i + 3i_1 = 70 \quad$ ...(i)

Applying KVL in loop $BCDEB$

$-5(i - i_1) - 90 + 6i_1 = 0$

$-5i + 11i_1 = 90 \quad$ ...(ii)

Solving (i) & (ii)

$i_1 = 10A$

29.(B) $R_{cu} = \dfrac{1.7 \times 10^{-8} \times 0.25}{3 \times 10^{-6}}$

$R_{Al} = \dfrac{2.6 \times 10^{-8} \times 0.25}{3 \times 10^{-6}}$

$= 1.42\ m\Omega$

$= 2.12 m\Omega$

$R_{eq} = \dfrac{R_{cu}\ R_{Al}}{R_{cm} + R_{AL}} = 0.85\ m\Omega$

30.(15) $\left(\dfrac{E}{5+r}\right)5 = 1.25 \quad \Rightarrow \quad \dfrac{E}{5+r} = 0.25$

$\left(\dfrac{E}{2+r}\right)^2 = 1 \quad \Rightarrow \quad \dfrac{E}{2+r} = 0.5$

$\dfrac{5+r}{2+r} = \dfrac{0.5}{0.25} = 2 \quad \Rightarrow \quad r = 1\Omega$

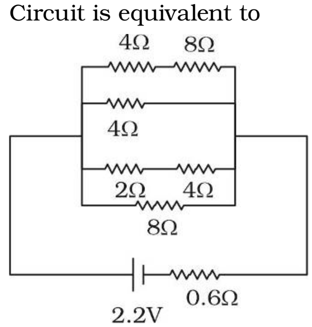
$\Rightarrow \quad E = 1.5V$

31.(B) $\dfrac{1}{R_{eq}} = \dfrac{A}{\rho_1 l} + \dfrac{A}{\rho_2 l}$

$\dfrac{1}{R_{eq}} = \dfrac{\pi d^2}{4l}\ \dfrac{\rho_1 + \rho_2}{\rho_1 \rho_2}$

$$l = \frac{\pi d^2 R_{eq}(\rho_1 + \rho_2)}{4\rho_1\rho_2} = \frac{3.14 \times 4 \times 10^{-6} \times 3 \times 63}{4 \times 12 \times 51 \times 10^{-8}} = 97m$$

32.(B) Circuit is equivalent to



$R_{eq} = 1.6 + 0.6 = 2.2\Omega$

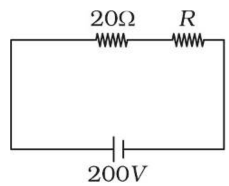
$P = \dfrac{V^2}{R_{eq}} = 2.2W$

33.(D) Option $4 \to 2\Omega$ and 4Ω in parallel $= \dfrac{2 \times 4}{6} = \dfrac{4}{3}$

$R_{eq} = \dfrac{4}{3} + 6 + 8 = \dfrac{46}{3}\Omega$

34.(B) 500 W, 100 V

$R = \dfrac{10^4}{500} = 20\Omega$

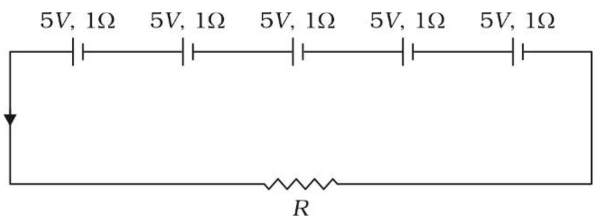


$\left(\dfrac{200}{20+R}\right)^2 \times 20 = 500$

$\dfrac{200}{20+R} = 5$

$R = 20\Omega$

35.(C)



$i = \dfrac{5+5+5+5+5}{5+R}$

If the cells are connected in parallel combination

$E_{eq} = \dfrac{\frac{5}{1}+\frac{5}{1}+\frac{5}{1}+\frac{5}{1}+\frac{5}{1}}{1+1+1+1+1} = 5v$

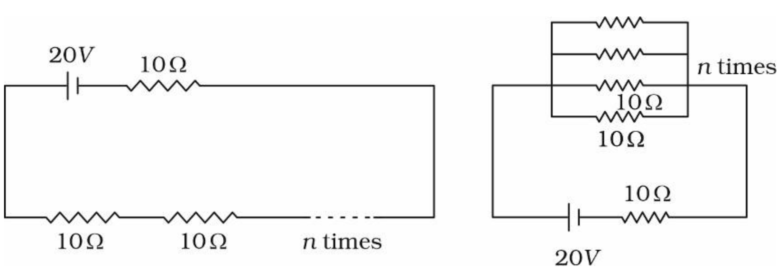
$r_{eq} = \dfrac{1}{1+1+1+1+1} = \dfrac{1}{5}\Omega$

$\therefore \qquad i' = \dfrac{5}{\frac{1}{5}+R}$

Now $i = i'$ $\qquad \Rightarrow \qquad 1 + R/5 = \dfrac{1}{5} + R$

Or $R = 1\Omega$

36.(20)



$I = \dfrac{20}{10n+10}$

$I' = \dfrac{20}{\frac{10}{n}+10}$

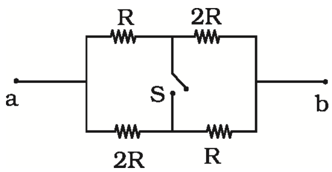
$I' = 20I$

$\Rightarrow \qquad \dfrac{20 \times n}{10+10n} = 20\left(\dfrac{20}{10n+10}\right) \qquad \Rightarrow \qquad n = 20$

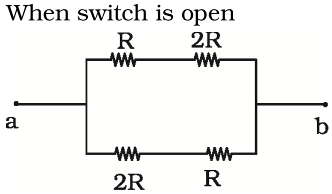
37. (B) As per color coding mnemonic:

$R = 7500 \pm 5\% = \left(7500 \pm 375\right)\Omega$

38.(9)

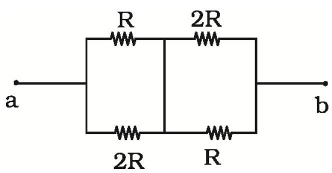


When switch is open



$R_{ab} = \dfrac{3R}{2}$

When switch is closed

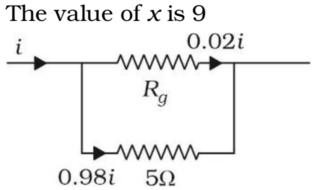


$$R_{ab} = \frac{R \times 2R}{2R + R} + \frac{2R \times R}{2R + R} = \frac{4R}{3}$$

$$\frac{\left(R_{ab}\right)_{open}}{\left(R_{ab}\right)_{closed}} = \frac{\frac{3R}{2}}{\frac{4R}{3}} = \frac{9}{8}$$

The value of x is 9

39.(D)

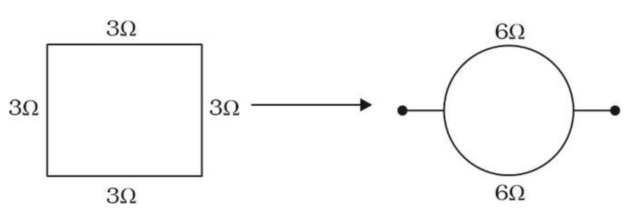


$(0.02i) \times R_g = (0.98i) \times 5$

$2R_g = 98 \times 5$

$R_g = 245\,\Omega$

40.(3)

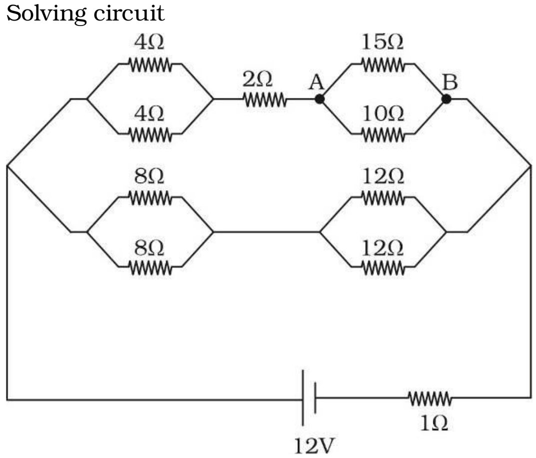


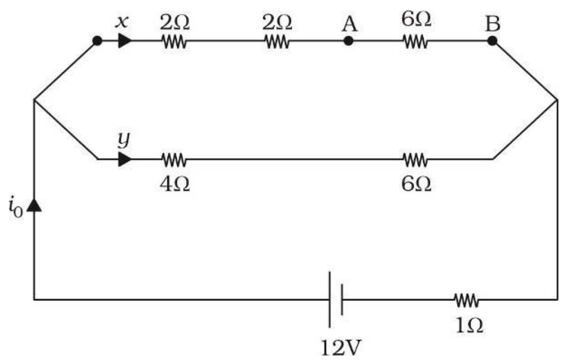
As we know $R \propto L$

$R_{eq} = 3\Omega$

41.(6) we have to find the voltage difference $V_A - V_B = ?$

Solving circuit





On solving above circuit

$i_0 = 2A$

$x = 1A$

$y = 1A$

$V_{AB} = i_{AB} \cdot R_{AB}$

$= x \cdot 6$

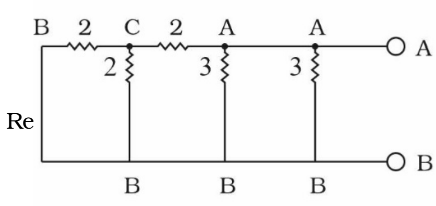
$V_{AB} = 6V$

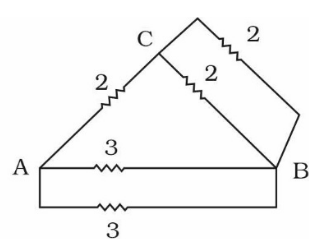
42.(4) In case 1, $P_1 = \dfrac{V^2}{R}$

In case 2, $P_2 = 2 \times \dfrac{V^2}{R/2}$

$P_1 : P_2 = 1 : 4$

43.(A)

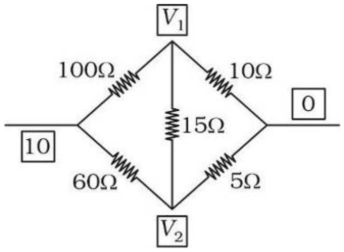




44.(3840) $\dfrac{H_2}{H_1} = \left(\dfrac{I_2}{I_1}\right)^2 \cdot \left(\dfrac{R_2}{R_1}\right)\left(\dfrac{t_2}{t_1}\right)$

$$H_2 = \left(\frac{8}{4}\right)^2 \left(\frac{R}{R}\right)\left(\frac{5}{1}\right)(192)J = 3840J$$

45.(D)



$$\frac{10-V_1}{100}+\frac{V_2-V_1}{15}+\frac{0-V_1}{10}=0 \qquad ......(1)$$

$$\frac{10-V_2}{60}+\frac{V_1-V_2}{15}+\frac{0-V_2}{5}=0 \qquad ......(2)$$

Solving (1) and (2)

$V_1 = 0.865\ V$ and $V_2 = 0.792\ V$

Current $= \dfrac{V_1 - V_2}{R} = 4.87\ mA$

46.(B) $H = i^2RT$

$500 = (1.5)^2 R \times 20 \qquad$ ... (i)

When $i = 3A$

$H_2 = (3)^2 \times R \times 20 \qquad$ ... (ii)

$$\frac{(2)}{(1)} \Rightarrow \frac{H_2}{500} = \frac{(3)^2 \times R \times 20}{(1.5)^2 \times R \times 20} = 4 \quad \Rightarrow \quad H_2 = 2000J$$

47.(2500) $H = i^2Rt$; $\quad 10 \times 10^{-3} = \left(2 \times 10^{-3}\right)^2 R \times 1$

$$\Rightarrow \qquad R = \frac{10}{4} \times 10^3 = 2500\Omega$$

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17.(3) Current in the potentiometer wire, $i = \dfrac{E}{r_1 + R_0 + \dfrac{R_0}{2}} = \dfrac{E}{r_1 + \dfrac{3R_0}{2}}$

For null point potential difference across the cell must be $\dfrac{E}{2}$.

$$\Rightarrow -ir_1 + E - \left(\frac{l}{100}\right)R_0 i = \frac{E}{2} \qquad \Rightarrow \frac{E}{2} = i\left(r_1 + \frac{l}{100}R_0\right)$$

$$\Rightarrow \frac{E}{2} = \frac{E}{r_1 + \dfrac{3R_0}{2}}\left(r_1 + \frac{72}{100}R_0\right) \qquad \Rightarrow \frac{1}{2} = \frac{2}{2r_1 + 3R_0}\left(r_1 + \frac{18}{25}R_0\right)$$

$$\Rightarrow 2r_1 + 3R_0 = 4r_1 + \frac{72}{25}R_0 \qquad \Rightarrow 3(50) = 2r_1 + \frac{72}{25}(50)$$

$$\Rightarrow 150 - 144 = 2r_1 \qquad \Rightarrow r_1 = 3\Omega$$

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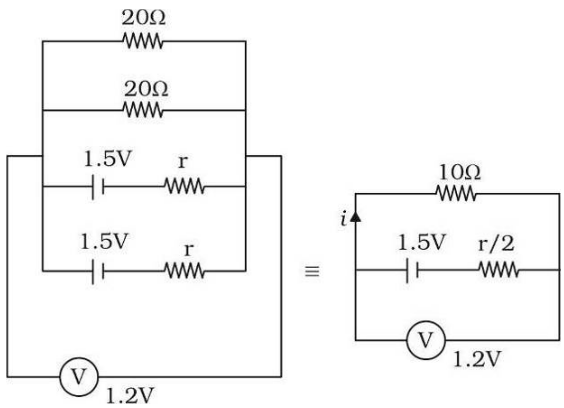
1.(C)



$$i=\frac{1.5}{10+\frac{r}{2}}=\frac{3}{r+20}$$

But $\Rightarrow v=10i \quad \Rightarrow\ 1.2=10\left(\frac{3}{r+20}\right) \qquad \Rightarrow\ r+20=\frac{300}{12}=25 \qquad \Rightarrow\ r=5\Omega$

2.(25) $\frac{e_1}{e_2}=\frac{l_1}{l_2}$

$\frac{3}{2}=\frac{75}{l_2}$

$l_2=50\,cm$

$l_1-l_2=75-50=25\,cm$

3.(B) A and B will be parallel in series with C $\Rightarrow\ R_{eq}=\frac{2\times 4}{2+4}+6=\frac{8}{3}+6=\frac{22}{3}$

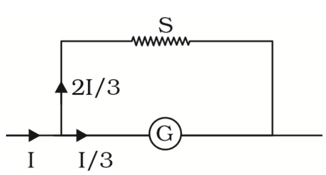
4.(25) $I=\frac{25}{R+R_w}=\frac{25}{20+30}=\frac{1}{2}Amp$

Y = potential by radiant $=\frac{iR}{L}=\frac{1}{2}\times\frac{20}{10}=1V/m$

balancing length = 2.5 m

$E=y\times$ balancing length $=1\times 2.5=2.5m$

5.(B)



$\frac{2I}{3}S=\frac{I}{3}G \Rightarrow G=2S$

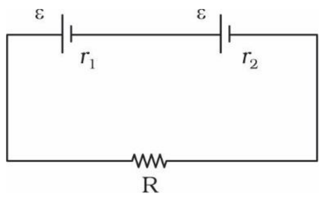
6.(450) $(2)^2 R(15) = 300 \Rightarrow R = 5\Omega$

Energy developed $= (3)^2 (5)(10) = 450J$

7.(2) $R_{eq} = 2.5\Omega$

$i = \dfrac{V}{R_{eq}} = 2A$

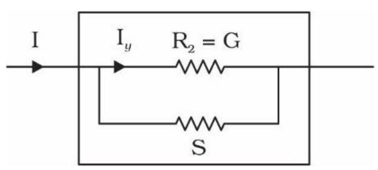
8.(A)



$V = \varepsilon - Ir_2 = 0$

$\varepsilon = \dfrac{2\varepsilon r_2}{(r_1 + r_2 + R)} \Rightarrow R = r_2 - r_1$

9.(D)



Coil expirimence deflection θ

$I = K\theta$

Where K = figure of merit

$I_y = Kn$

$\dfrac{IGS}{G+S} = I_y(G)$

$I = I_y \dfrac{(G+S)}{S} \Rightarrow \dfrac{nK(G+S)}{S}$

10.(300) As the length is doubled, area of cross section is halved (volume constant)

$R' = \dfrac{\rho(2l)}{(A/2)} = 4\left(\dfrac{\rho l}{A}\right) = 4R$

Thus, new resistance is 4 times old resistance

% increase $= \left(\dfrac{4R - R}{R}\right) \times 100 = 300\%$

11.(C) $R = \dfrac{\rho\ell}{A}; R = \dfrac{\rho\ell^2}{V}$; $\dfrac{\Delta R}{R} = \dfrac{2\Delta L}{L}$; $\dfrac{\Delta R}{R} \times 100 = \dfrac{2\Delta L}{L} \times 100 = 0.8\%$

12.(C) $r_1 = R_1 + R_2 = 10\ \Omega$

$r_2 = r_1 \,||\, R_3 = 5\ \Omega$

$r_3 = r_2 + R_4 = 10\ \Omega$

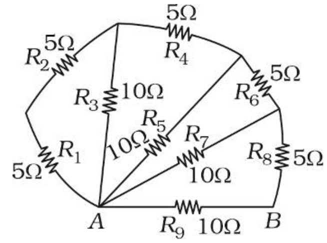
$r_4 = r_3 \,||\, R_5 = 5\ \Omega$

$r_5 = r_4 + R_6 = 10\ \Omega$

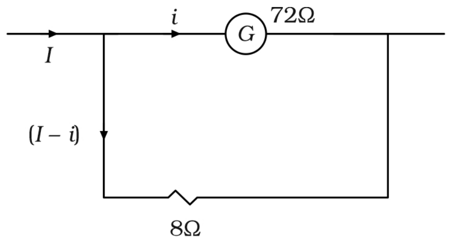
$r_6 = r_5 \,||\, R_7 = 5\ \Omega$

$r_7 = r_6 + R_8 = 10\ \Omega$

$r_8 = r_7 \,||\, R_9 = 5\ \Omega$



13.(B)

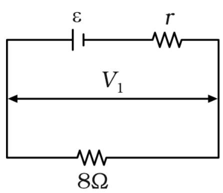


$72i = 8(I - i)$

$\Rightarrow \quad i = \dfrac{8I}{80} = \dfrac{I}{10}$

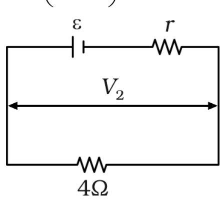
Percentage of total current $= \dfrac{\frac{I}{10}}{I} \times 100 = 10\%$

14.(8)



$V_1 = 3.PG$

$\varepsilon - \left(\dfrac{\varepsilon}{r+8}\right) r = 3PG$

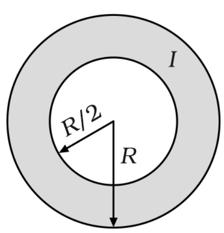


$V_2 = 3.PG$

$\varepsilon - \left(\dfrac{\varepsilon}{r+4}\right) r = 2PG \qquad \Rightarrow \dfrac{1 - \dfrac{r}{r+8}}{1 - \dfrac{r}{r+4}} = \dfrac{3}{2} \quad \Rightarrow \dfrac{(r+8-r)}{r+8} \times 2 = \left(\dfrac{r+4-r}{r+4}\right) 3$

$\Rightarrow 16(r+4) = 12(r+8) \quad \Rightarrow r = 24 - 16 \quad \Rightarrow r = 8\Omega$

15.(48) $R = 4mm$



$j = 4 \times 10^6 A/m^2$

$I = j\pi\left(R^2 - \dfrac{R^2}{4}\right)$

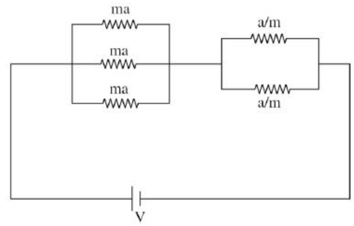
$I = 4 \times 10^6 \times \pi \times \dfrac{3}{4} \times \left(4 \times 10^{-3}\right)^2$

$I = 48\pi A$

16.(12) Current is outer part from $\dfrac{r}{2}$ to r = $j\left(\pi r^2 - \dfrac{1}{4}\pi r^2\right) = j\dfrac{3}{4}\pi r^2 = 1 \times 10^6 \times \dfrac{3}{4} \times \pi \times \left(4 \times 10^{-3}\right)^2 = 12\pi A$

$\Rightarrow \quad x = 12$

17.(3) $R_{eq} = \frac{ma}{3} + \frac{a}{2m}$



$\frac{dR_{eq}}{dm} = \frac{a}{3} - \frac{a}{2m^2}$

For R_{eq} to be minimum $\frac{dR_{eq}}{dm} = 0$

$m = \sqrt{\frac{3}{2}} \qquad \Rightarrow \qquad x = 3$

18.(10) Whetstone Bridge arrangement:

$\frac{R}{3} = \frac{4}{6} \Rightarrow R = 2\Omega$

$\therefore\ R_{eq} = \frac{6 \times 9}{6+9} = \frac{54}{15} \qquad \therefore\ i = \frac{36}{54/15} = \frac{36}{54} \times 15 = 10A$

19.(B) $mgH = P \times N$

$50\% \times \frac{9 \times 10^4}{60 \times 60} \times 40 \times 10 = P \times N$

$\frac{9 \times 10^4 \times 2}{36} = 100 \times N$

$N = 50$

20.(A) $2 = R_0\left(1 + \alpha\left(10 - T\right)\right)$

$3 = R_0\left(1 + \alpha\left(30 - T\right)\right)$

For absolute temperature T = 0°C

$\frac{2}{3} = \frac{1+10\alpha}{1+30\alpha} \Rightarrow 2 + 60\alpha = 3 + 30\alpha$

$30\alpha = 1$

$\alpha = \frac{1}{30} = 0.033°C^{-1}$

21.(8) $R_{eq} = R\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right]$

$R_{eq} = R\left[\frac{8+4+2+1}{8}\right]$

$R_{eq} = \frac{15R}{8}$

$R = 1\Omega$

$I = \frac{V}{R_{eq}}$

$I = \frac{3}{15R/8}$

$I = \frac{3 \times 8}{15R} = \frac{8}{5R}$

$I = \frac{8}{5 \times 1} = \frac{8}{5}$

$I = \frac{a}{5}$

$a = 8$

22.(15) If the Heat is H.

$$H=\left(\frac{V^2}{R_1}\right)(20) \text{ and } H=\left(\frac{V^2}{R_2}\right)60$$

where R_1 & R_2 are resistances of two coils.

When both coils are used.

$$\left(\frac{V^2}{R_1}+\frac{V^2}{R_2}\right)t=H \Rightarrow \left(\frac{H}{20}+\frac{H}{60}\right)t=H \Rightarrow t=\frac{60}{4}=15 \text{ minutes}.$$

23.(144) $R=\text{Slope}=\tan 45°=1$

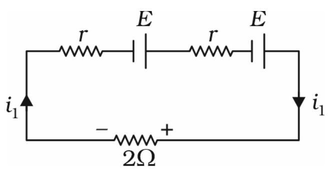
$$R=\frac{\rho\ell}{A} \Rightarrow \rho=\frac{RA}{\ell}=\frac{3.14\times(1.2\times10^{-2})^2}{31.4}$$

$= 144\times10^{-5}\Omega-m = 144\times10^{-3}\ \Omega-cm$

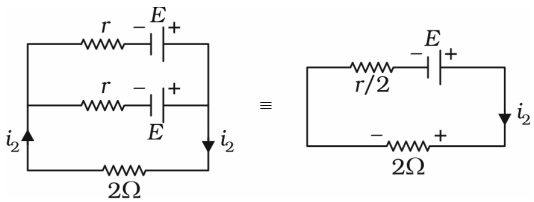
24.(10) $i=\frac{15}{3}=5A$

$V_B-V_A=15-5\times1=10V$

25.(A)



$$i_1=\frac{2E}{2r+2}=\frac{E}{r+1}$$

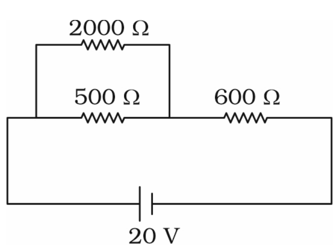


$$E_{eq}=\frac{rE+rE}{r+r};\quad E_{eq}=E$$

$$i_2=\frac{E}{2+\frac{r}{2}} \quad \because \quad i_1=i_2 \ ; \quad \frac{E}{r+1}=\frac{E}{2+\frac{r}{2}};\quad 2+\frac{r}{2}=r+1$$

$$1=\frac{r}{2};\quad r=2\Omega$$

26.(8)



$$R_{eq}=600+\frac{500\times2000}{500+2000}=1000\Omega$$

$$i=\frac{20}{1000}=\frac{1}{50}A \quad \therefore \quad V=iR_{VR}=\frac{1}{50}\times400=8V$$

$R_{vR}=R_{eq}$ of voltmeter and 500Ω

27.(136)

$$J = \frac{E}{\rho}$$

$$\frac{I}{A} = \frac{E}{\rho}$$

$$\frac{1}{2\times10^{-6}} = \frac{E}{1.7\times10^{-8}}$$

$$E = \frac{1.7}{2}\times10^{-2}\frac{V}{m}$$

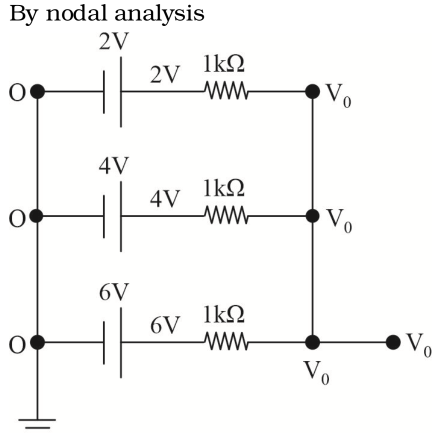
$$F = eE$$

$$F = 1.6\times10^{-19}\times\frac{1.7}{2}\times10^{-2} = 0.8\times10^{-21}\times1.7$$

$$F = 1.36\times10^{-21}$$

$$F = 136\times10^{-23}N$$

28.(4) By nodal analysis



$$\frac{V_0-2}{1}+\frac{V_0-4}{1}+\frac{V_0-6}{1}=0$$

$3V_0 = 12 \Rightarrow V_0 = 4\,volts$

29.(4) Each wire has resistance $= \rho\frac{4\ell}{\pi d^2} = r$

Eight wire in parallel, then equivalent resistance is

$$\frac{r}{8} = \frac{\rho\ell}{2\pi d^2}$$

Single copper wire of length $2l$ has resistance

$$R = \rho\frac{2\ell\times4}{\pi d_1^2} = \frac{\rho\ell}{2\pi d^2}$$

$\Rightarrow\ d_1 = 4d$

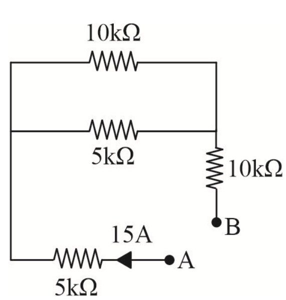
30.(D) Equivalent resistance between

A and B is $= \left(\frac{10}{3}+15\right)k\Omega$

$= \frac{55}{3}k\Omega$

Potential difference between the points

A and B $= 15mA\times\frac{55}{3}k\Omega$ = 275 V



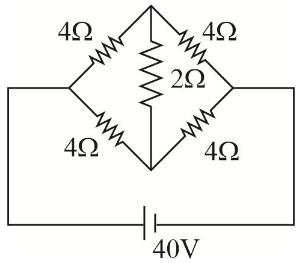
31.(18) $E \propto \ell$

$\dfrac{E_1}{E_2} = \dfrac{\ell_1}{\ell_2}$; $\dfrac{1.2}{1.8} = \dfrac{36}{\ell_2}$; $\ell_2 = 54\,cm$

$\Delta\ell = 54 - 36 = 18\,cm$

32.(A) $R_1 = 4 + 4 = 8\Omega$

$R_2 = 4 + 4 = 8\Omega$



$\dfrac{1}{R_{eq}} = \dfrac{1}{8} + \dfrac{1}{8}$

$\dfrac{1}{R_{eq}} = \dfrac{2}{8}$

$R_{eq} = \dfrac{8}{2}$

$\boxed{R_{eq} = 4\Omega}$

$I = \dfrac{V}{R_{eq}} = \dfrac{40}{4} = 10$

$\boxed{I = 10\ ampere}$

33.(20) $\dfrac{P}{Q} = \dfrac{40}{(100-40)} \Rightarrow \dfrac{P}{Q} = \dfrac{2}{3}$ ...(1)

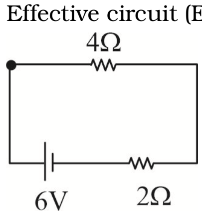
$\dfrac{P+x}{Q} = \dfrac{80}{(100-80)} \Rightarrow \dfrac{P+x}{Q} = \dfrac{80}{20}$ ...(2) $P + x = 4Q$

From, eq. (1) and (2). $P + x = \dfrac{4 \times 3P}{2}$

$P + x = 6P$

$5P = x \Rightarrow \boxed{x = 20\Omega}$

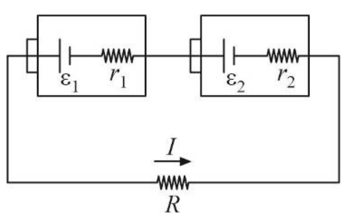
34.(A) Effective circuit (External circuit is Balanced wheat stone bridge)



35.(20) E = Potential gradient × null point length

$20 \times 10^{-3} = \dfrac{4 \times R}{(780 + R) \times 300} \times 60$; Solving above equation $R = 20\Omega$

36.(A)



$I = \dfrac{2\varepsilon_0}{R + r_1 + r_2}$

Given, $\varepsilon_0 - Ir_1 = 0$ $\Rightarrow$ $\varepsilon_0 - \dfrac{2\varepsilon_0 r_1}{R + r_1 + r_2} = 0$ $\Rightarrow$ $R + r_1 + r_2 = 2r_1$ $\Rightarrow$ $R = r_1 - r_2$

37.(2400)

$$\frac{R}{100-l}=\frac{S}{l} \quad ; \quad \frac{5.6\ k\Omega}{70}=\frac{S}{30}$$

$$S=\frac{3}{7}\times 5.6$$

$$S=2.4\ k\Omega$$

$$S=2400\ \Omega$$

38.(B) $v_d=\frac{eE}{m}\tau$

As $T\uparrow \Rightarrow \tau\downarrow$

So with temp $V_d\downarrow$ now

$I=neAV_d$

$$V_d=\frac{I}{neA}=\frac{V}{RneA} \quad ; \quad V_d=\frac{V}{\rho Lne} \quad ; \quad V_d\propto\frac{1}{L}$$

39.(40)

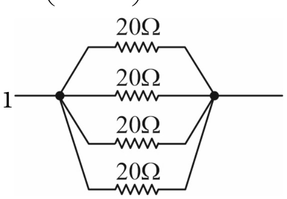
40.(A) Let initial length is L_0

Then, final length becomes $L_0+2L_0=3L_0$

As $R=\rho\frac{L}{A}=\rho\frac{L^2}{V}$ (where $V=vol^m$ of wire)

So $R\propto L^2$; $\frac{R_{New}}{R_{original}}=\frac{(3L_0)^2}{(L_0)^2}=\frac{9}{1}$

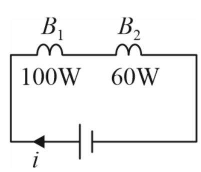
41.(780) $\left(\frac{4}{R+20}\right)\times 20\times\frac{60}{300}=20\times 10^{-3} \Rightarrow R=780\Omega$

42.(C) Cell 1 

$R_{net}=\frac{20}{4}=5\Omega$

Cell 2 $\frac{H_1}{H_2}=\frac{\frac{V^2}{R_1}}{\frac{V^2}{R_2}}=\frac{R_2}{R_1}=\frac{2}{3}$

43.(14) $P_{100}=I^2R_{100}$ . . . . (i)



$P_{60}=I^2R_{60}$ . . . . (ii)

$I=\frac{220}{R_{60}+R_{100}}$ . . . . (iii)

From (i) and (iii)

$$P_{100}=\left(\frac{220}{R_{60}+R_{100}}\right)^2 R_{100}$$

here $R_{60}=\frac{(220)^2}{60}$; $R_{100}=\frac{(220)^2}{100}$

Solving $P_{100}\simeq 14.06$

44.(B) $R_1 + R_2 = \rho_1 \frac{\ell}{A} + \rho_2 \frac{\ell}{A}$

$= \frac{\ell}{\sigma_1 A} + \frac{\ell}{\sigma_2 A}$

$\frac{\ell}{A}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right) = \frac{1}{\sigma}\frac{2\ell}{A}$

$\frac{\sigma_1 + \sigma_2}{\sigma_1 \sigma_2} = \frac{2}{\sigma}$

$\sigma = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$

45.(2) $\frac{1}{R_{eq}} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$

$\therefore \quad R_{eq} = 3\Omega$

$I = \frac{6V}{3\Omega} = 2A$

46.(A) Constantan and manganin are used to make standard resistance coils as they have high resistivity and low temperature coefficient of resistance.

47.(B) Let length of X be x

& length of Y be y

also $\quad R = \frac{\rho l}{A} \times \frac{l}{l} = \frac{\rho l^2}{volume} \quad \Rightarrow \quad R \propto l^2 \quad \Rightarrow \quad R = kl^2$

Wire W has length $2x$

$R_w = k(2x)^2$

Also, $\quad R_w = 2R_y = 2ky^2$

Thus, $\quad k4x^2 = 2ky^2 \qquad \mu_1 \simeq \frac{9}{7} \quad \frac{x}{y} = \frac{1}{2}$

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Solutions of Archive - JEE Main & Advanced

Capacitors	Class - XII | Physics

JEE Main 2021

1.(D) Series connection

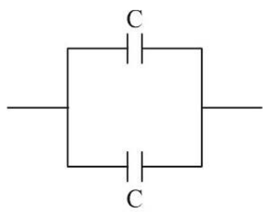
$$C_{equialent(i)} = \frac{C}{2}$$

Case (i)

Parallel connection

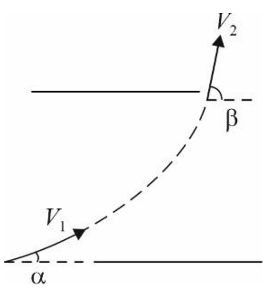
$$C_{equialent(ii)} = 2C$$

$$\frac{C_{(i)}}{C_{(ii)}} = \frac{C/2}{2C} = \frac{1}{4}$$



Case (ii)

2.(A)



Because electric field is velocity perpendicular to it is constant.

So, $V_1 \cos\alpha = V_2 \cos\beta$

So, $\frac{KE_1}{KE_2} = \frac{\frac{1}{2} m V_1^2}{\frac{1}{2} m V_2^2}$

$$= \left(\frac{V_1}{V_2}\right)^2 = \frac{\cos^2\beta}{\cos^2\alpha}$$

3.(Bonus)

$$C_1 + C_2 = \frac{15}{4}\frac{C_1C_2}{C_1+C_2}$$

$$(C_1 + C_2)^2 = \frac{15}{4}C_1C_2$$

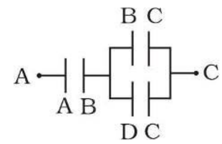
$$\frac{C_1^2}{C_1C_2} + \frac{C_2^2}{C_1C_2} + \frac{2C_1C_2}{C_1C_2} = \frac{15}{4}$$

$$\frac{1}{x} + x + 2 = \frac{15}{4} \quad \Rightarrow \quad \frac{x^2+1}{x} = 7/4 \quad \Rightarrow \quad 4x^2 + 4 = 7x \quad \Rightarrow \quad 4x^2 - 7x + 4 = 0$$

$D = 49 - 64 < 0$; No real Answer

4.(2) $l = 2cm$

$b = \frac{3}{2}cm$



$\Rightarrow \quad \frac{x \in_0}{d} = \frac{C.2C}{C+2C}$

$\Rightarrow \quad \frac{x \in_0}{d} = \frac{2}{3}C \quad \Rightarrow \quad \frac{x \in_0}{d} = \frac{2}{3}.\frac{\in_0}{d}lb \Rightarrow x = \frac{2}{3}lb$

$\therefore \quad x = \frac{2}{3} \times 2 \times \frac{3}{2}cm^2 = 2cm^2$

5.(864) $q = CV$

$q = 14 \times 10^{-12} \times 12$ Coulomb

$q = 168 \times 10^{-12}$ Coulomb

When dielectric is inserted, charge remains same.

The mechanical energy with which the dielectric oscillates is $U_f - U_i$

$U = \frac{q^2(k-1)}{2\,kC}$

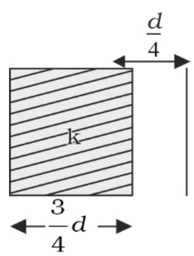
$U = \frac{168 \times 10^{-12} \times 168 \times 10^{-12}(7-1)}{2 \times 7 \times 14 \times 10^{-12}}$

$U = 12 \times 10^{-12} \times 12 \times 6J$

$U = 864 \times 10^{-12} J$

$U = 864 pJ$

6.(A)



$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$

Where $C_1 = \frac{K\varepsilon_0 A}{\frac{3d}{4}}$

$C_2 = \frac{\varepsilon_0 A}{\frac{d}{4}}$

$\frac{1}{C'} = \frac{3d}{4K\varepsilon_0 A} + \frac{d}{4\varepsilon_0 A}$

$C' = \frac{4K\varepsilon_0 A}{d(3+K)} = \frac{4K}{3+K}C_0$

7.(16) $Q_1 = 10 \times 2 = 20\mu C, Q_2 = 0$

Charge will be distributed in proportion to capacitance

$q_2 = \frac{C_2}{C_1 + C_2}.(Q_1 + Q_2) = \frac{8}{10} \times 20 = 16\ \mu C$

8.(161) $C_{eq} = \dfrac{\varepsilon_0 A}{d_1 + \dfrac{d_2}{k}} = \dfrac{8.85 \times 100}{5 + 0.5} = \dfrac{885}{5.5} \cong 161$

9.(2) Initial $\Delta V_C = \dfrac{q}{C} = 10 \text{ Volts} = \Delta V_R$

$\dfrac{\Delta V_R}{R} = 2\ \mu A$

10.(3) $C = \dfrac{\epsilon_0\ A}{\left(\dfrac{d_1}{k} + d_2\right)} = \dfrac{\epsilon_0 \times 2}{\left(\dfrac{0.5}{3.2} + 0.5\right)} \approx 3 \epsilon_0$

11.(B) $\dfrac{1}{C_1} = \displaystyle\int_0^{d/2} \dfrac{dx}{(\epsilon_0 + kx)A} = \dfrac{1}{KA}\left[\log_e(\epsilon_0 + kx)\right]_0^{d/2} = \dfrac{1}{KA}\log_e\left(\dfrac{\epsilon_0 + kd/2}{\epsilon_0}\right)$

$\dfrac{1}{C_2} = \displaystyle\int_{d/2}^{d} \dfrac{dx}{\left[\epsilon_0 + k(d-x)\right]A} = \dfrac{1}{-KA}\left[\log(\epsilon_0 + k(d-x))\right]_{d/2}^{d} = \dfrac{-1}{KA}\log_e\left(\dfrac{\epsilon_0}{\epsilon_0 + kd/2}\right) = \dfrac{1}{KA}\left(\dfrac{\epsilon_0 + kd/2}{\epsilon_0}\right)$

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{2}{KA}\log_e\left(\dfrac{\epsilon_0 + kd/2}{\epsilon_0}\right)$

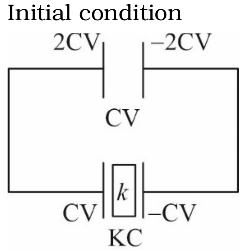
$\therefore C_{eqv.} = \dfrac{KA}{2\log_e\left(\dfrac{\epsilon_0 + kd/2}{\epsilon_0}\right)} = \dfrac{KA}{2\log_e\left(\dfrac{2\epsilon_0 + kd}{2\epsilon_0}\right)}$

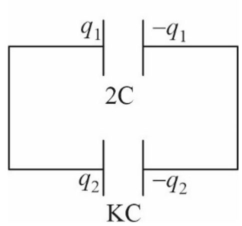
12.(A) $E_{\text{inside}} = \dfrac{E_{\text{outside}}}{K}$

$E_{\text{inside}} = E_{\text{outside}} - E_{\text{induced}}$

$\dfrac{q_0}{KA\varepsilon_0} = \dfrac{q_0}{A\varepsilon_0} - \dfrac{q_b}{A\varepsilon_0}$ $\qquad q_b = q_0\left(1 - \dfrac{1}{k}\right)$

13.(D) Initial condition





From conservation of charge

$q_1 + q_2 = 3CV$

Let final potential be V'

$(KC + 2C)V' = 3CV$

$V' = \dfrac{3V}{K+2}$

14.(C) $C_1 = \dfrac{k\varepsilon_0 A}{d}$

$$C_2 = \frac{3k\varepsilon_0 A}{2d}$$

$$C_3 = \frac{5k\varepsilon_0 A}{3d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{k\varepsilon_0 A} + \frac{2d}{3k\varepsilon_0 A} + \frac{3d}{5k\varepsilon_0 A}$$

$$C_{eq} = \frac{15k\varepsilon_0 A}{34d}$$

15.(B) As we know,

$$q = q_0(1 - e^{-t/RC})$$

$$\Rightarrow \frac{q}{C} = \frac{q_0}{C}(1 - e^{-t/RC})$$

$$\Rightarrow V = V_0(1 - e^{-t/RC})$$

$$\Rightarrow 50 = 100(1 - e^{-t/RC})$$

$$\Rightarrow \frac{1}{2} = 1 - e^{-t/RC} \quad \Rightarrow e^{-t/RC} = \frac{1}{2}$$

$$\therefore t = RC\ln 2 = 10^2 \times 10^{-6} \times 0.69 = 0.69 \times 10^{-4} s$$

16.(A) $i = \frac{V}{R} = \frac{V}{\left(\rho d/A\right)} = \frac{k\varepsilon_0 A}{d}\frac{V}{\rho} \times \frac{1}{k\varepsilon_0}$

$$= \frac{CV}{k\varepsilon_0 . \rho} = \frac{2 \times 10^{-12} \times 40}{50 \times 8.85 \times 200} \times 10^{12} = 0.9 \times 10^{-3} A$$

17.(A) $\frac{1}{C_{1eq}} = \frac{1}{K_1}\frac{d/2}{\varepsilon_0 A/2} + \frac{d/2}{K_2 \varepsilon_0 A/2}, C_{2eq} = \frac{\varepsilon_0 A/2}{d}$ and $C_{eq} = C_{1eq} + C_{2eq}$

$$C_{eq} = \frac{K_1 K_2}{K_1 + K_2}\frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{2d} = \frac{\varepsilon . A}{d}\left(\frac{K_1 K_2}{K_1 + K_2} + \frac{1}{2}\right)$$

18.(A) $V_{AB} = E - ir$

$i = \frac{5}{5} = 1A$

$V_{AB} = 5 - 1 \times 1$

$= 4V$

$q_{AB} = C_{AB} V_{AB}$

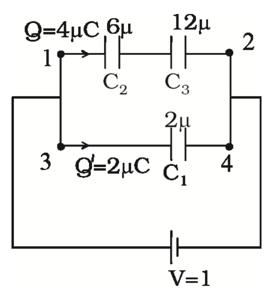
$C_{AB} = \frac{4 \times 4}{4 + 4} = 2\mu F$

$q_{AB} = 2\mu F \times 4 = 8\mu C$

Charge of $4\mu F = 8\mu C$

19.(A) $Q_1 : Q_2 : Q_3 = 2 : 4 : 4 = 1 : 2 : 1$

Let $V = 1$

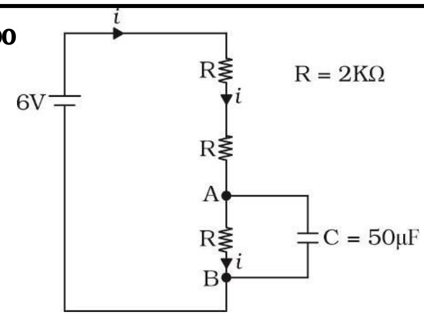


$C_{eq}\left(in\, brach\, 1 - 2\right) = \frac{6 \times 12}{18}\mu F = 4\mu F$

$\Rightarrow Q_2 = Q_3 = 4\mu C$

$\Rightarrow C_{eq}\left(in\, branch\, 3 - 4\right) = 2\mu F \qquad \Rightarrow Q_1 = 2\,\mu C$

20.(100



Current through capacitor will be zero

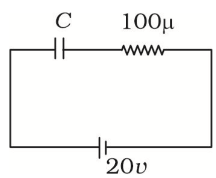
$i = \dfrac{6}{3R} = 10^{-3} A$

$\therefore\ V_{AB} = iR = 2V$

$\therefore\ Q = CV = 50\mu F \times 2V$

$Q = 100\mu C$

21.(B) Here $V_C = \dfrac{q}{C} = \varepsilon(1 - e^{-t/RC})$



$2 = 20\left(1 - e\dfrac{-10^{-6}}{RC}\right)$

Solving, $C = 0.95\mu F$

22.(4) $C = 200\mu F$

$V = 200\ volts$

$K = 2$

$\Delta U = \dfrac{1}{2}CV^2(K-1) = \dfrac{1}{2} \times 2 \times 10^{-4} \times 4 \times 10^{4} J = 4J$

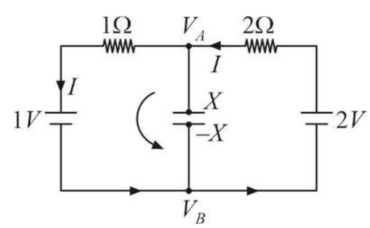
JEE Advanced 2021

1.(1.33) At steady state

At steady state current passing through capacitor is 0

Then current in circuit $I = \dfrac{2-1}{3}$

$I = \dfrac{1}{3} Amp$



Potential at point $A = V_A$

Potential at point $B = V_B$

$V_A - 1\left(\dfrac{1}{3}\right) - 1 = V_B$

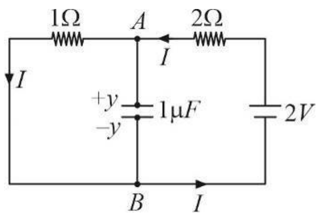
$V_A - V_B = 1 + \dfrac{1}{3} = \dfrac{4}{3}V$

Charge on capacitor $Q = C(V_A - V_B)$

$$X = 1 \times \frac{4}{3} \mu C$$

$$X = 1.33\ \mu C$$

2.(0.67)



At steady state current through capacitor = 0

$$I = \frac{2}{3} Amp$$

Potential difference across capacitor = Potential difference across $1\ \Omega$ resistor

$$V_A - V_B = I(1) = \frac{2}{3} V$$

Charge on Capacitor $= y = C(V_A - V_B)$

$$= 1 \times \frac{2}{3} \mu F = 0.67\ \mu F$$

Solutions of Archive - JEE Main & Advanced

Capacitors	Class - XII | Physics

JEE Main 2022

1.(A) $u_i = \frac{1}{2}(10C)V^2$

$u_f = \frac{1}{2}(15C)V^2$ Where C is the capacitance without dielectric

% change $= \frac{u_f - u_i}{u_i} \times 100 = 50\%$

2.(A)

$$\begin{array}{|ccc|} + & & - \\ + & \vec{E} & - \\ + & & - \end{array} \; E = \frac{\sigma}{\varepsilon_0}$$

$F = q\frac{\sigma}{\varepsilon_0} = 10N$

If one plate is removed

$$\begin{array}{|l} + \\ + \\ + \rightarrow \frac{\sigma}{2\varepsilon_0} \\ + \\ + \end{array}$$

$F' = \frac{q\sigma}{2\varepsilon_0} = \frac{F}{2}$

$F' = \frac{10}{2} = 5N$

3.(D) Dielectric strength = Maximum electric field between capacitor plates that a dielectric could bear a without any break down

$3.6 \times 10^7 = \frac{\sigma}{K \in_0} = \frac{Q}{KA \in_0}$

$3.6 \times 10^7 = \frac{7 \times 10^{-6}}{K(30\pi \times 10^{-4})} \in_0$

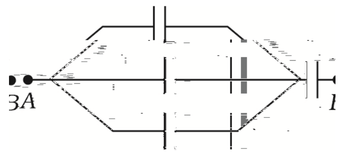
$3.6 \times 10^7 = \frac{7 \times 10^{-6}}{K \times 10^{-4} \times 7.5(4\pi \in_0)}$

$K = \frac{7 \times 10^{-6} \times 9 \times 10^9}{3.6 \times 10^7 \times 10^{-4} \times 7.5} = \frac{63}{3.6 \times 7.5} = 2.33$

4.(A) $v = \frac{q^2}{2C}$ $\Rightarrow$ $v \propto q^2$ $\Rightarrow$ $q_f = 1.2q$

$\Rightarrow$ $q_f - q_i = 2$ $\Rightarrow$ $1.2q - q = 2$ $\Rightarrow$ $q = 10C$

5.(6)



$C_{eq} = 6\mu F$

6.(A) Initial capacitance $= \dfrac{\epsilon_0 A}{d}$

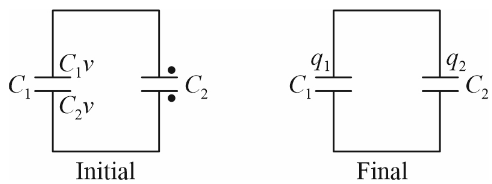
As there will be no electric field inside metal sheet hence effective distance between plates will be reduced by thickness of metal sheet

New capacitance $= c' = \dfrac{\epsilon_0 A}{d - d/2}$

$C' = \dfrac{2 \epsilon_0 A}{d} = 2C_0$

$\therefore \dfrac{C'}{C_0} = 2$

7.(A)



$C_1V = q_1 + q_2$; $\dfrac{q_1}{C_1} = \dfrac{q_2}{C_2}$

$C_1v = q_2 \dfrac{C_1}{C_2} + q_2$; $q_2 = \dfrac{C_1C_2v}{C_1 + C_2}$

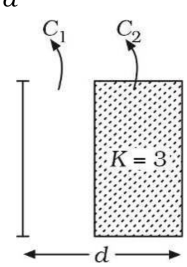
8.(C) When put in electric field, both center displace a little & a dipole moment arises.

9.(A) $\dfrac{1}{C_{eq}} = \dfrac{1}{10} + \dfrac{1}{15} + \dfrac{1}{20} = \dfrac{6+4+3}{60} = \dfrac{13}{60}$

$\therefore \quad C_{eq} = \dfrac{60}{13}\ \mu F$

$\therefore \quad Q = C_{eq} \times V = \dfrac{60}{13} \mu F \times 13V = 60\ \mu C$

10.(C) $C_0 = \dfrac{A\varepsilon_0}{d} = 4\mu F$ (Given)



$C_1 = \dfrac{A\varepsilon_0}{d/2} = 8\ \mu F$

$C_2 = \dfrac{KA\varepsilon_0}{d/2} = 3 \times 2 \times 4\ \mu F = 24\ \mu F$

$C_{eq} = \dfrac{C_1C_2}{C_1 + C_2} = \dfrac{8 \times 24}{32} = 6\,\mu F$

11.(125) $C_1 = 50 \times 10^{-12} F$ $V_1 = 100V$

$C_2 = 50 \times 10^{-12} F$ $V_2 = 0$

$V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$ $\Rightarrow$ $V = \dfrac{50 \times 100 + 0}{100}$

$V = 50V$

Loss $= U_i - U_f$

$= \dfrac{1}{2}C \times 100^2 - 2\left(\dfrac{1}{2}C \times 50^2\right)$

$= \dfrac{1}{2}C\left[10000 - 5000\right]$

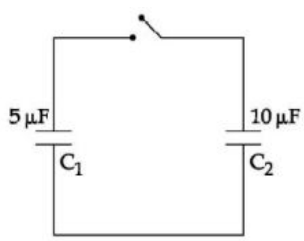
Loss $= \dfrac{1}{2} \times 50 \times 10^{-12} \times 5000 \times 10^{+9} nJ$

Loss $= 125 nJ$

12.(23) All plates are in parallel connection

$$C_{eq} = \frac{\varepsilon_0 A}{5b} + \frac{\varepsilon_0 A}{3b} + \frac{\varepsilon_0 A}{b} = \frac{\varepsilon_0 A}{b}\left(\frac{3+5+15}{15}\right) = \frac{23}{15}\frac{\varepsilon_0 A}{15}$$

13.(100) $Q_i = 5\mu F \times 30V$



$Q_i = 150\mu C$ ...(1)

After closing switch : Lets assume final potential to be V.

$Q_f = (15)V$ ...(2)

$150\mu C = 15V$

Final charge on $C_2 = 10\mu F \times 10 = 100\mu C$

14.(D) $\because$ $E = \dfrac{Q^2}{2C}$ $\therefore$ $E = E_0 e^{-2t/\tau}$

$\dfrac{E_0}{2} = E_0 e^{-2t_1/\tau}$

$\ell n2 = \dfrac{2t_1}{\tau}$ ... (i)

$Q = Q_0 e^{-t/\tau}$; $\dfrac{Q_0}{8} = Q_0 e^{-t_2/\tau}$

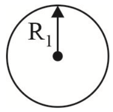
$\ell n8 = \dfrac{t_2}{\tau}$

$3\ell n2 = \dfrac{t_2}{\tau}$,,, (ii)

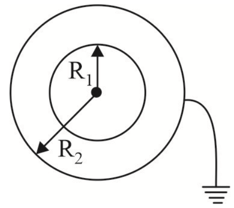
$\dfrac{(i)}{(ii)}$; $\dfrac{1}{3} = \dfrac{2t_1}{t_2}$; $\dfrac{t_1}{t_2} = \dfrac{1}{6}$

15.(A) $Q = CV$ $\Rightarrow$ $V = \dfrac{Q}{C}$ $\Rightarrow$ $V \propto Q$ $\Rightarrow$ slope $= \dfrac{1}{C} = \dfrac{1}{2 \times 10^{-6}} = 5 \times 10^5$

16.(A) Here $C_1 = 4\pi\varepsilon_0 R_1$



Now new capacitance of the system as shown in figure



$$= \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

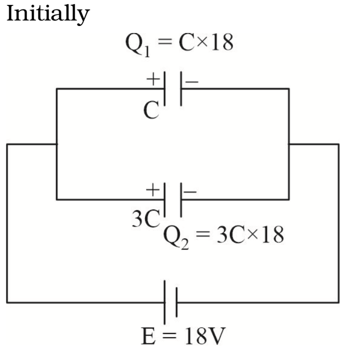
Given that

$$\frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1} = n \times 4\pi\varepsilon_0 R_1$$

$$\frac{R_2}{R_2 - R_1} = n$$

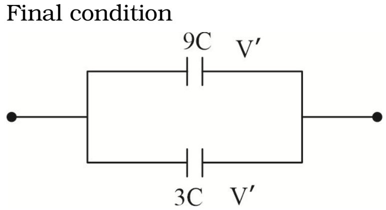
$$\frac{R_2}{R_1} = \frac{n}{n-1}$$

17.(6) Initially



Total charge on plates $Q = Q_1 + Q_2$

Final condition



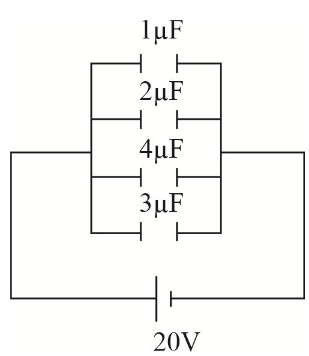
Charge conservation

$$C \times 18 + 3C \times 18 = 9CV' + 3CV'$$

$$4 \times 18 = 12V'$$

$$V' = 6V$$

18.(A) $C_{eq} = 1 + 2 + 4 + 3$



$C_{eq} = 10\mu F$

$C_{eq} = 10 \times 10^{-6} F$

$q = C_{eq} V$

$q = 10 \times 10^{-6} \times 20$

$q = 200 \times 10^{-6}$ coulomb

$\boxed{q = 200\mu C}$

19.(60) The capacitor shown in figure is made up of two capacitor C_1 and C_2 connected in series.

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A}{t_1}, \quad C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A}{t_2}$$

Since C_1 *and* C_2 are in series so charges on both capacitor is same.

$\boxed{Q_1 = Q_2}$

$C_1(100 - V) = C_2V$ V = voltage of foil

$\frac{\varepsilon_0 \varepsilon_{r_1} A}{t_1}(100 - V) = \frac{\varepsilon_0 \varepsilon_{r_2} A}{t_2}V$

$\frac{3}{0.5}(100 - V) = \frac{4}{1}V$

$\frac{3}{0.5}(100 - V) = \frac{4}{1}V$

$300 - 3V = 2V$

$300 = 5V$

$V = 60$ volt

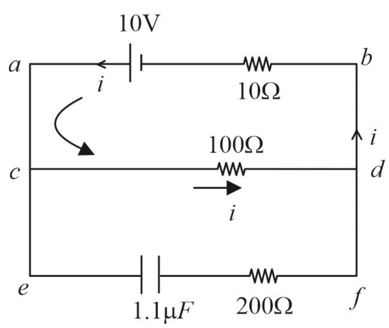
20.(C) $E_1 = \frac{1}{2}.2CV^2$

$E_2 = \frac{1}{2}(5C)V^2 + \frac{1}{2}.\frac{(CV)^2}{5C} = \frac{1}{2}.\frac{26CV^2}{5}$

(Voltage const) (Charge const.)

$\frac{E_1}{E_2} = \frac{5}{13}$

21.(10)

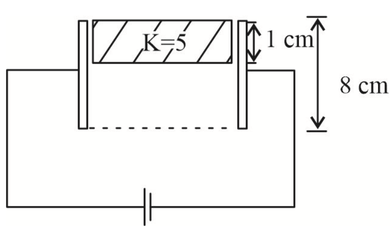


$i = \frac{10}{100 + 10} = \frac{1}{11}A$

$\therefore$ P.D across the capacitor $= V_e - V_f = V_c - V_d = i \times 100 = \frac{100}{11} volts$

$\therefore$ charge on capacitor, $Q = CV = 1.1 \times 10^{-6} \times \frac{100}{11} = 10 \times 10^{-6}C$

22.(240)



$C_{eqv.} = C_1 + C_2 = \frac{k \in_0 A_1}{d} + \frac{\in_0 A_2}{d}$

$= \frac{5 \times 8.85 \times 10^{-12} \times 4 \times 1 \times 10^{-4}}{4 \times 10^{-3}} + \frac{8.85 \times 10^{-12} \times 7 \times 4 \times 10^{-4}}{4 \times 10^{-3}}$

$$= \frac{8.85 \times 10^{-12}}{4 \times 10^{-3}} [20 + 28] \times 10^{-4} = 1.2 \in_0$$

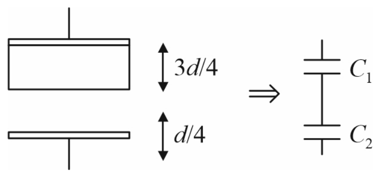
$$\therefore \text{ Energy stored } = \frac{1}{2} C_{eqv.} V^2 = \frac{1}{2} \times (1.2 \in_0) \times (20)^2 = 240 \in_0$$

23.(A) As per given condition,

$$24 = \frac{(40)(40k)}{(40 + 40k)}$$

Gives $k = 1.5$

24.(A) Here $C_1 = \frac{4k\varepsilon_0 A}{3d}$, $C_2 = \frac{4\varepsilon_0 A}{d}$



$$C_{net} = \frac{C_1 C_2}{C_1 + C_2} = \frac{4kC_0}{3 + K}$$

25.(C) $V = \frac{\frac{q_1 - q_2}{2}}{C} = \frac{q_1 - q_2}{2C}$

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1.(B) $F = qvB = \dfrac{qpB}{m} \Rightarrow F \propto \dfrac{q}{m}$ where p = momentum

$\therefore$ Force ratio = $\dfrac{e}{1} : \dfrac{e}{2} : \dfrac{2e}{4} = 2 : 1 : 1$

Speed $\propto \dfrac{p}{m}$ $\quad\therefore$ Speed ratio $= \dfrac{1}{1} : \dfrac{1}{2} : \dfrac{1}{4} = 4 : 2 : 1$

2.(B) $B = \dfrac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}} \Rightarrow B \propto \dfrac{1}{\left(R^2 + x^2\right)^{3/2}}$ $\qquad \therefore \dfrac{B_1}{B_2} = 8 = \dfrac{\left\{R^2 + (0.2)^2\right\}^{3/2}}{\left\{R^2 + (0.05)^2\right\}^{3/2}}$.

Take cube root, and squaring $(2)^2 = \left(\dfrac{R^2 + 0.04}{R^2 + 0.0025}\right)$

$\Rightarrow \qquad 4R^2 + 0.01 = R^2 + 0.04 \Rightarrow 3R^2 = 0.03 \Rightarrow R^2 = 0.01 \qquad \therefore R = 0.1m$

3.(C) $B = \mu_0 n I \mu_r$

$\Rightarrow \qquad B = 4\pi \times 10^{-7} \times 500 \times 10^3 \times 5T$

$\therefore B = \pi T$

4.(D) $B = \dfrac{\mu_0 I}{4\pi R} + \dfrac{\mu_0 I}{4R} + \dfrac{\mu_0 I}{4\pi R} = \dfrac{\mu_0 I}{4\pi R}(2 + \pi)$

5.(C) Conceptual

6.(None)

Statements B, C, E are correct. So, none of the given options are correct.

7.(C) As we know, $r = \dfrac{mV}{qB}$

$\therefore r = \dfrac{\sqrt{2mk}}{qB} \quad \therefore \dfrac{r_p}{r_\alpha} = \dfrac{\dfrac{\sqrt{2m_p k_p}}{1 \cdot B}}{\dfrac{\sqrt{2m_\alpha k_\alpha}}{2 \cdot B}} \qquad \therefore \dfrac{2}{1} = \dfrac{2\sqrt{m_p k_p}}{\sqrt{m_\alpha k_\alpha}} \quad \therefore 1 = \dfrac{m_p k_p}{m_\alpha k_\alpha}$

$\therefore \dfrac{k_p}{k_\alpha} = \dfrac{m_\alpha}{m_p} = \dfrac{4}{1} = 4 : 1$

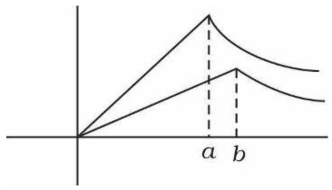
8.(D) Work done by magnetic field (magnetic force) on moving charge $= 0 \left[\because \vec{F} \perp \vec{V}\right]$

9.(D)

Current per unit area

$\dfrac{I}{\pi a^2} \qquad \dfrac{I}{\pi b^2}$

At any x less than radius



$$B(x)=\frac{\mu_0 \frac{I}{a^2}\times x^2}{2\pi x} \qquad B(x)=\frac{\frac{\mu_0 I}{b^2}x^2}{2\pi x}$$

Slope (1) > slope (2)

10.(D) $R=\frac{mV}{qB}=\frac{\sqrt{2m(KE)}}{qB}$

$\therefore \quad \frac{r_d}{r_\alpha}=\sqrt{\frac{m_d(K.E.)}{m_\alpha(K.E.)}}\frac{q_\alpha}{q_d}$

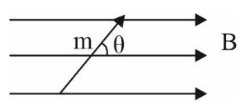
$\frac{r_d}{r_\alpha}=\left(\sqrt{\frac{2m}{4m}}\right)\left(\frac{2e}{e}\right)$ or $\sqrt{2}$

11.(D) $\frac{m_1}{m_2}=1 \qquad \frac{q_1}{q_2}=\frac{1}{2} \qquad \frac{v_1}{v_2}=\frac{2}{3}$

$R=\frac{mv}{qB} \Rightarrow \frac{R_1}{R_2}=1\times\frac{2}{3}\times\frac{1}{1/2}$

$R_1 : R_2 = 4:3$

12.(8) Given,



$m=9.85\times10^{-2}=\pi^2\times10^{-2}$

$T=\frac{5}{10}=\frac{1}{2}\text{sec}$

$\tau=mB\sin\theta=mB\theta$ for small θ

Also, $\tau=I\alpha$

$mB\theta=I\alpha$

$\alpha=\frac{mB\theta}{I}\Rightarrow$ implies $\omega^2=\frac{mB}{I}$

$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{I}{mB}}$

$T=2\pi\sqrt{\frac{5\times10^{-6}}{\pi^2\times10^{-2}\times B}}$

$\frac{1}{2}=\frac{2\pi}{\pi}\sqrt{\frac{5\times10^{-4}}{B}}\Rightarrow B=16\times5\times10^{-4}=8\,mT$

13.(B) $B=\frac{\mu_0 Nia^2}{2(a^2+r^2)^{3/2}}=\frac{\mu_0 Ni}{2a}\left(1+\frac{r^2}{a^2}\right)^{-3/2}$

as $r \ll a$, $\quad B=B_0\left(1-\frac{3}{2}\frac{r^2}{a^2}\right)$

$\frac{B-B_0}{B_0}=-\frac{3}{2}\frac{r^2}{a^2}$

14.(543) In each revolution $\Delta E = 2eV$

$\therefore$ In N revolution $\Delta E = 2N\ eV = \frac{1}{2}mv^2 = \frac{1}{2}m_p\left(\frac{C}{6}\right)^2$

$$N = \frac{m_p C^2}{4eV \times 36} = \frac{1.67 \times 9 \times 10^5}{4 \times 12 \times 36 \times 1.6} = 543.62$$

15.(3) $\vec{\tau} = \vec{m} \times \vec{B}$ where $m = \frac{\sqrt{3}}{4}a^2 i = \frac{\sqrt{3}}{4}\frac{1}{100}\frac{1}{5}$

$B = 20 \times 10^{-3}T \quad \theta = 90°$

$\tau = mB\sin\theta = \sqrt{3} \times 10^{-5}Nm \quad \Rightarrow \quad x = 3$

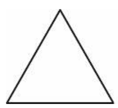
16.(B) $R = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB}$

Where k is kinetic energy

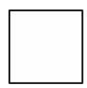
$\therefore \quad \frac{R_1}{R_2} = \sqrt{\frac{m_1}{m_2}}\frac{q_2}{q_1} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$

$R_1 < R_2 \quad \therefore$ Lighter ion will be deflected more

17.(3)



Side length = $3a$
No. of turns = 8

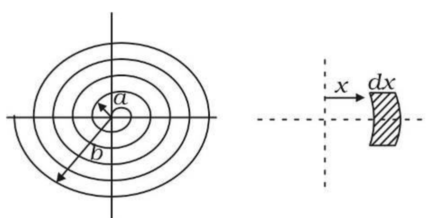


Side length = $4a$
No. of turns = 6

$\mu_1 = 8 \times i \times \frac{\sqrt{3}}{4}a^2 \qquad \mu_2 = 6 \times i \times a^2$

$\frac{\mu_1}{\mu_2} = \frac{2\sqrt{3}\,i\,a^2}{6\,i\,a^2} = \frac{1}{\sqrt{3}} \qquad y = 3$

18.(D)



$$\frac{dN}{dx} = \frac{N}{(b-a)}$$

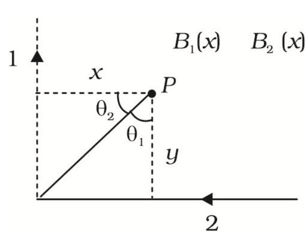
$$B = \int_a^b \frac{\mu_0 dNi}{2x} = \int_a^b \frac{\mu_0\left(\frac{N}{b-a}\right)dxi}{2x} = \frac{\mu_0 Ni}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

19.(C) $\vec{B}_P = \vec{B}_1 + \vec{B}_2$

$\frac{\mu_0 i}{4\pi x}\left(1+\sin\theta_2\right) \quad (x)$

$= \frac{\mu_0 i}{4\pi y}\left(1+\sin\theta_1\right) \quad (x)$

$B_P = \frac{\mu_0 i}{4\pi xy}\left(\sqrt{x^2+y^2} + (x+y)\right)$



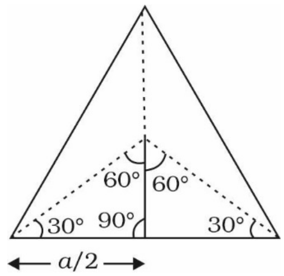
$\sin\theta_1 = \dfrac{x}{\sqrt{x^2+y^2}}$

$\sin\theta_2 = \dfrac{y}{\sqrt{x^2+y^2}}$

20.(B) $\dfrac{a/2}{d} = \tan 60°$

$d = \dfrac{a/2}{\tan 60} = \dfrac{a}{2\sqrt{3}}$

$d = \dfrac{a}{2\sqrt{3}} \times 10^{-2} m$



$B = 3 \times \left[\dfrac{\mu_0 i}{4\pi d}(\sin 60 + \sin 60)\right]$

$= 3 \times \dfrac{10^{-7} \times 1.5}{\dfrac{9}{2\sqrt{3}} \times 10^{-2}} \times \dfrac{2 \times \sqrt{3}}{2}$

$\dfrac{3 \times 10^{-7} \times 1.5 \times \sqrt{3} \times 2\sqrt{3}}{9 \times 10^{-2}} = 3 \times 10^{-5} T$

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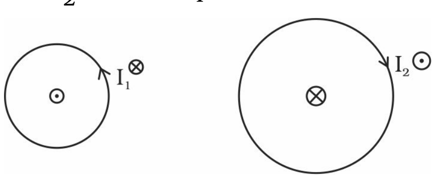
1.(4) $r = \dfrac{\sqrt{2mqV}}{qB} \Rightarrow r \propto \sqrt{\dfrac{m}{q}}$

$\Rightarrow \quad \dfrac{r_1}{r_2} = \sqrt{\dfrac{m_1}{m_2}\left(\dfrac{q_2}{q_1}\right)} = \sqrt{\dfrac{32}{4}\dfrac{(2)}{(1)}} = 4$

2.(AB) We can consider the two loops to behave as two bar magnets for analysing the magnetic field.

For $I_1 \rightarrow$ North pole is towards + z axis

For $I_2 \rightarrow$ North pole is towards – z axis



Since field will be $\perp_r$ to x-y plane, option A is correct

Since figure is symmetric, so B will depend on r.

Hence option B is correct.

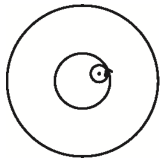
At center

$B_1 = \dfrac{\mu_0}{2\pi}\dfrac{I_1}{R} \odot$

$B_2 = \dfrac{\mu_0}{2\pi}\dfrac{I_0}{2R} \otimes$

$B_2 > B_1$ hence $\otimes$

Very close to B_1, field will point outwards.



It will be zero at some point.
Option C is incorrect.
In between magnetic field will be inwards
Hence option D incorrect.

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1.(A) In magnetic field force is perpendicular to velocity

So, $w = 0$

$\Delta K.E. = 0$

Both statement 1 and 2 is correct and 2 is correct explanation for statement 1.

2.(C) $B_{center} = \frac{\mu_0 I}{2r}$

So, $B = \frac{\mu_0 I}{2r}$ ...(i)

$\Rightarrow B_{axis} = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$

$= \frac{\mu_0 I r^2}{2\left(r^2 + \frac{r^2}{4}\right)^{3/2}}$

$= \frac{\mu_0 I r^2 4^{3/2}}{2\left(5r^2\right)^{3/2}}$

$= \frac{\mu_0 I r^2 \times 8}{2\ 5^{3/2}\ r^3}$

$B_{axis} = \frac{\mu_0 I}{2r}\left(\frac{2}{\sqrt{5}}\right)^3$ ...(ii)

$\Rightarrow (ii)/(i) \qquad \Rightarrow B_{axis} = \left(\frac{2}{\sqrt{5}}\right)^3$

3.(D) $r = \frac{mv}{qB} = \frac{\sqrt{2mK.E.}}{qB}$

$\therefore r \propto \frac{\sqrt{m}}{q} \qquad \Rightarrow r_p : r_d : r_a = \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$

4.(B) $B = \frac{\mu_0 I r}{2\pi R^2}$ inside the wire $(r < R)$

5.(A) $B \propto n\,i$

6.(C) $R = \frac{mv}{qB}; \qquad R_P = \frac{1 \times v}{1 \times B}$

$R_\alpha = \frac{4 \times v}{2 \times B} = \frac{2v}{B} \qquad \therefore \qquad \frac{R_\alpha}{R_P} = 2 : 1$

7.(5) $10^{-5}N = \dfrac{\mu_0 I^2}{2\pi d}\times 10\ cm,\ 10^{-5} = \dfrac{2\times10^{-7}\times(5)^2}{d}\times10^{-1}$

$d = 2\times25\times\dfrac{10^{-8}}{10^{-5}} \qquad \Rightarrow \qquad d = 50\times10^{-3}m$

$d = 50\times10^{-1}cm \qquad \Rightarrow \qquad d = 5\ m$

8.(2) Radius of charge particle $r = \dfrac{\sqrt{2mk}}{qB}$

For deuteron, $r_d = \sqrt{\dfrac{2(2m)k}{eB}}$; For proton, $r_p = \dfrac{\sqrt{2(m)k}}{eB}$

$r_d : r_p = \sqrt{2} : 1$; $x = 2$

9.(C) $\overrightarrow{B_1} = \dfrac{\mu_0}{4\pi}\dfrac{2I_1}{r_1}\left(-\hat{k}\right) = -2\times10^{-5}\hat{k}\,T$

$\overrightarrow{B_2} = \dfrac{2}{3}\times10^{-5}\hat{k}\,T \qquad \therefore \qquad \vec{B} = \overrightarrow{B_1}+\overrightarrow{B_2} = 10^{-5}\hat{k}\left[-2+\dfrac{2}{3}\right] = -\dfrac{4}{3}\times10^{-5}\hat{k}\,T$

$\overrightarrow{F_m} = q\left[\vec{V}\times\vec{B}\right] = 3\pi\left[\left(2\hat{i}+3\hat{j}\right)\times\left(\dfrac{4}{3}\times10^{-5}\right)\left(-\hat{k}\right)\right] = 4\pi\times10^{-5}\left[2\hat{j}-3\hat{i}\right] = 4\pi\times10^{-5}\left[-3\hat{i}+2\hat{j}\right]$

10.(10) $R = \dfrac{mv}{qB} = \dfrac{p}{qB} = \dfrac{\sqrt{2m\times K.E.}}{qB}$

$= \dfrac{\sqrt{2\times1.6\times10^{-27}\times24\times5\times10^{3}\times1.6\times10^{-19}}}{1.6\times10^{-19}\times0.5} = 2\times4.89\times10^{-2}m = 9.79\,cm = 10\,cm$

11.(D) For $r > R$ (outside point):

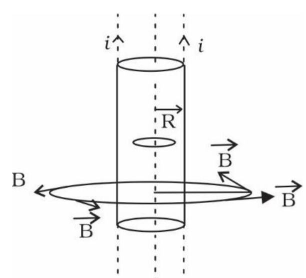
Using Ampere's law : $B(2\pi r) = \mu_0 i$

$B = \dfrac{\mu_0 i}{2\pi r}\propto\dfrac{1}{r}$

For $r < R$ (inside points):

$B(2\pi r) = \mu_0\times0$

$\Rightarrow B = 0$

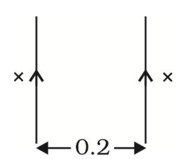


12.(C) $B = \dfrac{\mu_0 i}{2\pi d} = 2\times10^{-7}\times\dfrac{x}{0.2} = x10^{-6}$

$F = iBl = x\times x\times10^{-6}\times l$

$\dfrac{F}{l} = x^2\times10^{-6} = 2\times10^{-6}$

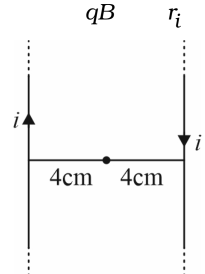
$x = \sqrt{2} = 1.4$



13.(C) $K.E_f = 4K.E_i \quad\Rightarrow\quad v_f = 2v_i$

As $r = \dfrac{mv}{qB} \Rightarrow \dfrac{r_f}{r_i} = \dfrac{2}{1}$

14.(B)



$B = B_1 + B_2$; $\quad B = 2\left(\frac{\mu_0 i}{2\pi r}\right)$

$300 \times 10^{-6} = 4\left(\frac{\mu_0}{4\pi}\right)\frac{i}{r}$

$3 \times 10^{-4} = 4 \times 10^{-7} \times \frac{i}{4 \times 10^{-2}}$

$3 \times 10^{-4} = 10^{-5} \times i$; $\quad i = 30A$

15.(C) Magnetic force $\perp$ velocity of particle

$\therefore$ Statement 2 : incorrect.

Statement 1 : correct

16.(C) Frequency $= \frac{qB}{2\pi m} = \frac{1.6 \times 10^{-19} \times 1 \times 10^{-4}}{2 \times \pi \times 9 \times 10^{-31}}$

$= \frac{160}{18\pi} \times 10^6\, Hz$

$= \frac{160 \times 7}{18 \times 22} \times 10^6\, Hz$

$= \frac{280}{99} \times 10^6\, Hz$

$= 2.8 \times 10^6\, Hz$

17.(B) Radius of circular path $R = \frac{\sqrt{2mk}}{qB}$

$q = \frac{\sqrt{2mk}}{RB}$

$\frac{q_1}{q_2} = \sqrt{\frac{m_1}{m_2}} \times \frac{R_2}{R_1} = \sqrt{\frac{9}{4}} \times \frac{5}{6} = \frac{5}{4}$

18.(B) $B_1 = 2 \times \frac{\mu_0 i}{2r} = \frac{\mu_0 i}{r}$

When it is rewound into 5 turns $[\because 2(2\pi r_1) = 5(2\pi r_2)]$

New radius $= \frac{2r}{5}$

So, Now $\frac{B_2}{B_1} = 5 \times \frac{\mu_0 i}{2\left(\frac{2r}{5}\right)} = \frac{25}{4}\frac{\mu_0 i}{r}$

So, $\frac{B_2}{B_1} = \frac{25}{4}$

19.(B) $\vec{B} = \left(2\hat{i} + 3\hat{j}\right)$

$\vec{a} = \left(\alpha\hat{i} - 4\hat{j}\right)$

$\vec{F} = q\left(\vec{V} \times \vec{B}\right)$

$\vec{F} \perp \vec{B}$

$\vec{a} \perp \vec{B}$

$\vec{a}.\vec{B} = O$

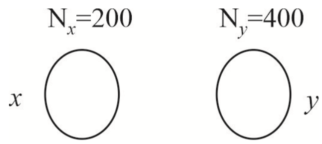
$\left(\alpha\hat{i} - 4\hat{j}\right).\left(2\hat{i} + 3\hat{j}\right) = 0$

$2\alpha - 12 = 0$

$2\alpha = 12$

$\alpha = 6$

20.(B) $B_X = \dfrac{\mu_0}{4\pi}\dfrac{N_x I}{R}.2\pi$



$B_x = \dfrac{\mu_0 N_x I}{2R}$

$B_y = \dfrac{\mu_0 N_y I}{2R}$ $[\because R_1 = R_2]$

$\dfrac{B_x}{B_y} = \dfrac{\mu_0 N_x I}{2R} \Big/ \dfrac{\mu_0 N_y I}{2R} = \dfrac{\mu_0 N_x I}{\mu_0 N_y I} = \dfrac{N_x}{N_y}$

$\dfrac{B_x}{B_y} = \dfrac{200}{400} = \dfrac{1}{2}$

21.(B) $\vec{M}_{net} = I\pi\left(r_2^2 - r_1^2\right)\left(-\hat{k}\right) = 7 \times \dfrac{22}{7} \times [(0.5)^2 - (0.3)^2](-\hat{k})$

solving $\vec{M}_{net} = \dfrac{-7\hat{k}}{2}$ $amp\ m^2$

22.(A) $\vec{E} = E\hat{k},\ \vec{B} = B\hat{j}$

$\vec{V} = V(\hat{i})$

$\vec{F}_{magnetic} = (-e)(V\hat{i} \times B\hat{j}) = eVB(-\hat{k})(along - z\, direction)$

$\vec{F}_{electrostatic} = (-e)E\hat{k} = eE(-\hat{k})(along - z\, direction)$

The given answer comes upon balancing electric and magnetic force but here both forces are along same direction.

So Lorentz force is not zero.

(There is error in giving directions of the field)

$V = \dfrac{E}{B}$ and $K = \dfrac{1}{2}mV^2$

$E = VB$

$E = \sqrt{\dfrac{2K}{m}} \times B$

$E = 192000V / m = 192\, kVm^{-1}$

23.(B) $R = \dfrac{mv}{qB} \Rightarrow v = \dfrac{qBR}{m}$ $\quad \therefore K.E. = \dfrac{1}{2}mv^2$

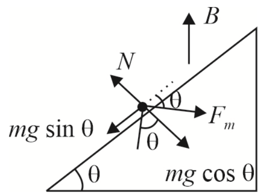
$= \dfrac{1}{2}m \times \left(\dfrac{qBR}{m}\right)^2 = \dfrac{\left(qBR\right)^2}{2m} = \dfrac{\left\{1.6 \times 10^{-19} \times 1 \times 60 \times 10^{-2}\right\}^2}{2 \times 1.6 \times 10^{-27}} = \dfrac{36 \times 1.6}{2} \times 10^{-13}\ Joules$

$= 18 \times \dfrac{1.6 \times 10^{-13}}{1.6 \times 10^{-19} \times 10^6} \times MeV = 18\, MeV$

24.(D) Current sensitivity $= \dfrac{\theta}{I} = \dfrac{NAB}{K}$

So $S \propto N$ and $S \propto \dfrac{1}{K}$

25.(A)



$F_m = ILB$

For minimum I,

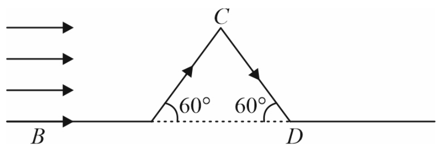
$F_m \cos\theta = mg \sin\theta$

$(I\,L\,B)\cos\theta = mg\sin\theta$

$I = \dfrac{mg\tan\theta}{LB}$

$I = \dfrac{(0.45)\times 10\times 1}{0.15} = 30A$

26.(C)



$\vec{F}_{CD} = \int i\,d\vec{\ell}\times\vec{B}$

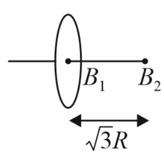
$= i\vec{\ell}\times\vec{B}$

$F_{CD} = i(\ell)B\sin 60°$

$= 10\left(\dfrac{5}{100}\right)(0.5)\times\dfrac{\sqrt{3}}{2} = .216$

27.(C) Have $B_1 = \dfrac{\mu_0 I}{2R}$

$B_2 = \dfrac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}} = \dfrac{\mu_0 IR^2}{2(8R^3)}$



Hence, $\dfrac{B_1}{B_2} = \dfrac{8}{1}$

28.(3) $B = \dfrac{n\mu_0 i}{2R} = \left(\dfrac{\mu_0}{4\pi}\right)\left(\dfrac{2\pi i}{R}\right)n$

$37.68\times 10^{-4} = \dfrac{10^{-7}\times(6.28i)\times 10^2}{5\times 10^{-2}}$

$i = 3A$

29.(11) $R = \dfrac{L}{2\pi} = \dfrac{314}{2\times 3.14}cm$

Or $R = 50\,cm$

$|\vec{M}| = i(\pi R^2)$

$= 14\times 3.14\times\left(\dfrac{50}{100}\right)^2$

$= 10.99$

$\simeq 11$

30.(A) Force on wire $Y = \dfrac{\mu_0 i_1 i_2}{2\pi d} \times l$

$$= \frac{4\pi \times 10^{-7} \times 3 \times 2}{2\pi \times 5 \times 10^{-2}} \times 0.5$$

$$= 12 \times 10^{-6}$$

$= 1.2 \times 10^{-5} N$ towards wire X